## Hybrid Parabolic Equation – Integral Equation Solvers for Analyzing Long Range Propagation Over Complex Terrain

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The analysis and design of long-haul, non-line-of-sight (NLOS) wireless links benefits from simulation tools that accurately account for both terrain and atmospheric features. At present, the channel characteristics of NLOS links are estimated using a wide variety of semi-empirical models, e.g. the Longley Rice (A. G. Longley et al, ESSA Tech. Rep. ERL 79-ITS 67, U.S. Government Printing Office) and the TIREM models (G. Benoit, ECAC-HDBK-86-076, DoD, ECAC). To enable modeling of site-specific effects, approximate electromagnetic simulation tools such as Ray tracing and Parabolic equations (PEs) (Levy, Parabolic Equation Methods for Electromagnetic Wave Propagation, The Institution of Engineering and Technology) are often used.

In particular, there is an extensive body of literature that applies PE methods to model propagation over rural, ocean, and urban environments. These methods can effectively handle simple terrain interfaces such as flat horizontal/vertical, perfectly conducting, or impedance surfaces, in addition to a variety of atmospheric/ionospheric conditions. Slanted interfaces oftentimes require staircase approximations, and modeling of scattering from realistic objects is fraught with difficulties. The latter can be simulated by modern, fast surface integral equation (IE) methods. Unfortunately, these methods do not permit modeling of atmospheric/ionospheric propagation effects (W. C. Chew et al, "Fast and Efficient Algorithms in Computational Electromagnetics," *Radio Sci. Meet.*).

This paper proposes a new framework for modeling NLOS propagation using a PE - IE hybrid. The framework's PE component simulates field propagation from the transmitting antenna through the atmosphere/ionosphere, and uses a wide-angle split-step Fourier method enhanced by Gabor-Daubechies sparsification techniques that produce significant computational savings over earlier PE solution methods. The framework's IE solver simulates field interactions with the terrain (ground, vegetation, buildings, etc.) and solves classical boundary integral equations through the application of the fast multipole method. Depending on the terrain characteristics, a variety of integral equations modeling impedance boundary conditions or penetrable interfaces are used.

The solver processes fields on "vertical" range-height slices extending from the ground/terrain into the atmosphere/ionosphere. Fields propagated from one slice to the next (denoted as input and output slices) consist of a direct and an indirect component. The direct component describes fields that do not interact with the terrain. This component is handled entirely by the PE solver, which propagates Gabor-Daubechies beamlets through the atmosphere/ionosphere while accounting for local disturbances in refractive index using phase screens. The indirect component describes fields that interact with the terrain between PE input and output slices. First, fields in the PE input slice are propagated to the terrain. Second, the IE solver computes equivalent sources that describe fields scattered from the terrain. Third, these scattered fields are propagated to the PE output slice. Steps one and three are performed efficiently using so-called fast far-field methods. The framework uses a forward-backward method to account for fields that bounce between terrain obstacles before reaching the receiver.

The above-described hybrid IE-PE solver has been successfully implemented in 2D, and 3D implementations are being developed. Numerical results demonstrating the memory and CPU efficiency of the solvers for a variety of terrain conditions will be provided at the conference.