Hybrid Parabolic Equation – Integral Equation Solvers for Analyzing Long Range Propagation Over Complex Terrain

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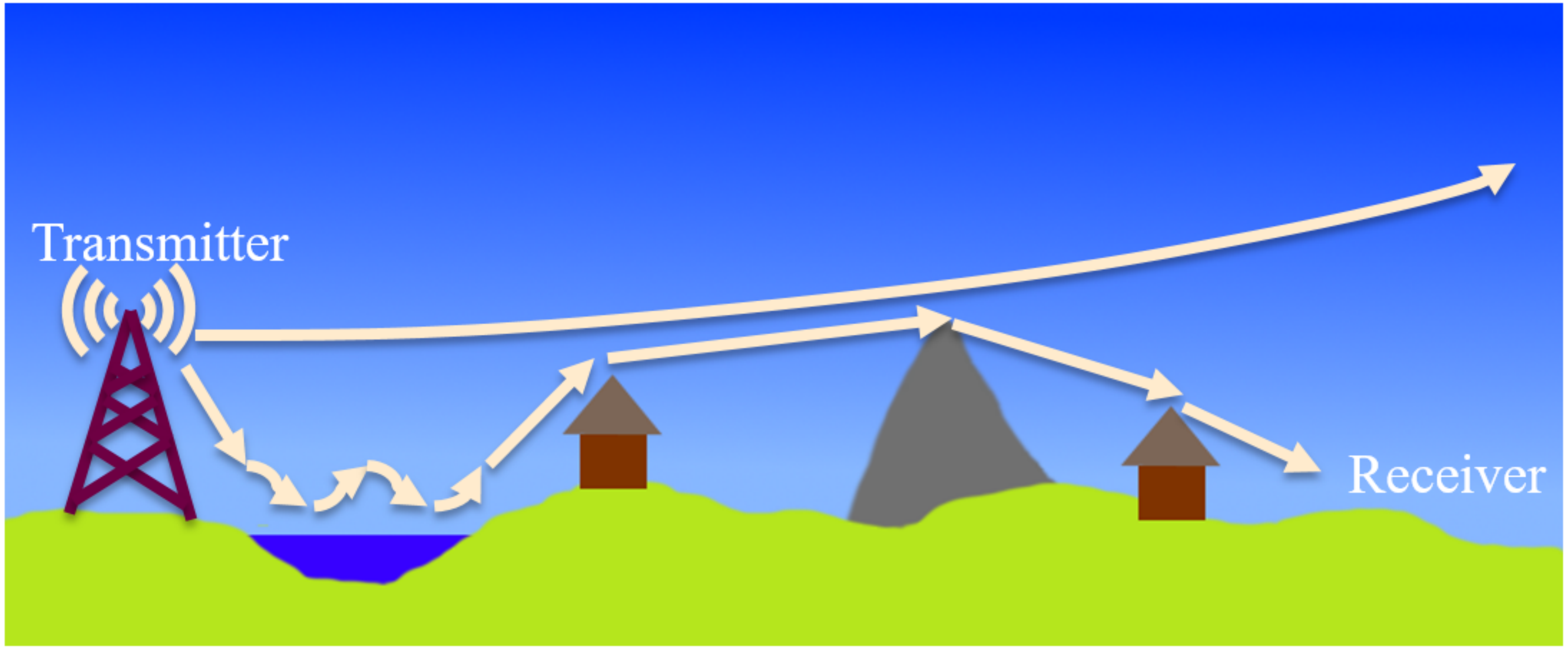
Propagation Over Rural Terrain



- Model long range propagation over wide frequency band (MHz -> GHz)
- Model complex terrain
- Model atmospheric effects, ducting, earth curvature

Old But Highly Relevant Problem…

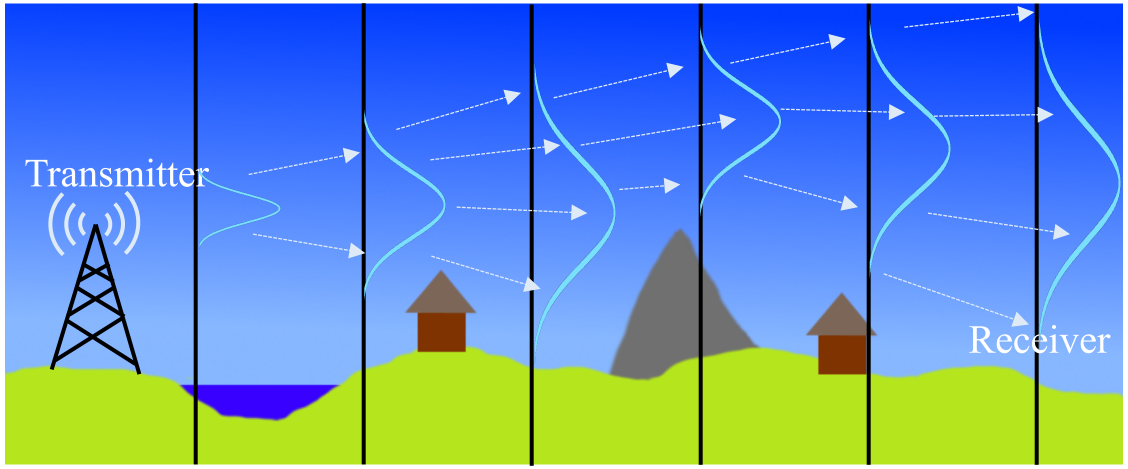
- **Longley Rice – TIREM models**
- **Asymptotic techniques: PO to Ray Tracing to GTD**
- **Parabolic Wave equations (PWE)**
- **Numerically rigorous CEM solvers – FDTD, integral Equations (IE)**

- **Key problems**
  - Scenes are electromagnetically HUGE
  - Scene is not deterministic

- **No single technique can capture all effects**

PWE : Basic Split-Step Scheme

- **Split range into vertical slices**
  1. Advance wavefront through each slice using spectral/plane wave propagator
  2. Apply phase screens to account for atmospheric disturbance



$$\nabla^2\varphi + k^2\varphi = 0$$

$$\varphi = \begin{cases} E & \text{H-pol} \\ H & \text{V-pol} \end{cases}$$

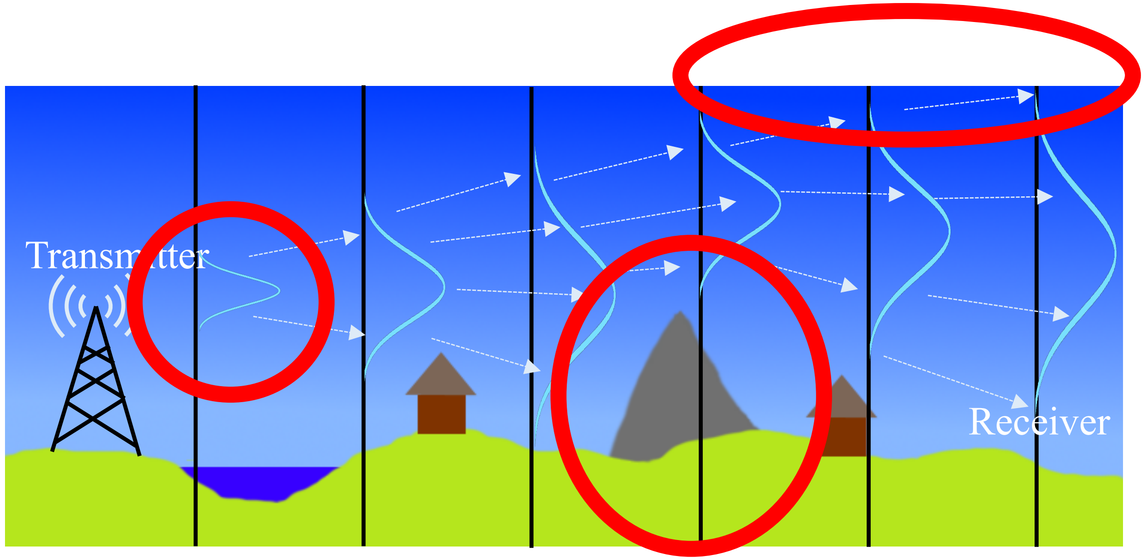
$$\varphi(x+\Delta x, z) = \mathrm{IFFT}\{\hat{\varphi}(x, k_z) e^{ik_0\sqrt{1-\frac{k_z^2}{k_0}}\Delta x}\} e^{-i\Delta x k_0 (n(z)-1)}$$

PWE : Weaknesses of Basic Split-Step Scheme

A. **Memory consumption / computational complexity**
   - High even for structured ray-like fields

B. **Upper Domain truncation**
   - Awkward implementation of PMLs, other RBCs

C. **Terrain modeling**
   - Staircase approximations or piecewise linear models
   - Poor material modeling capabilities

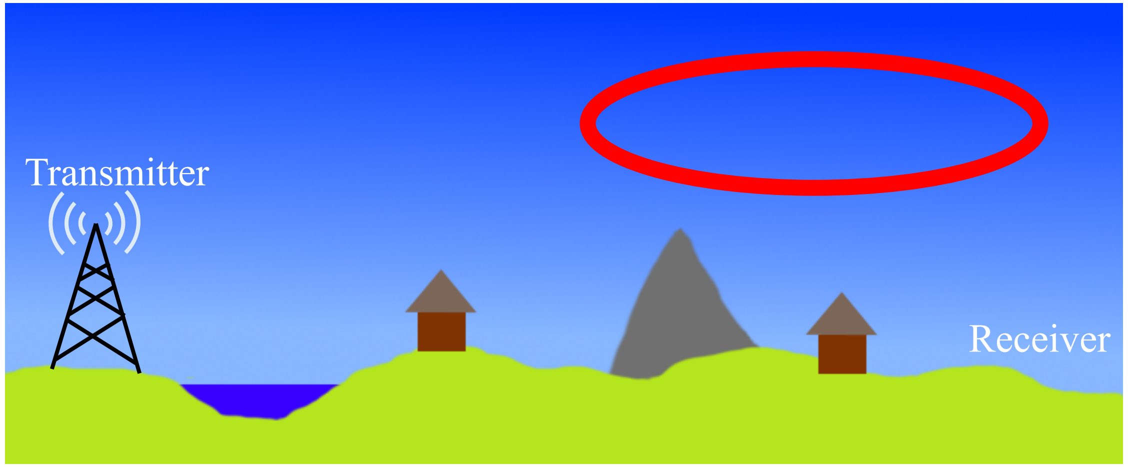


- Issues A and B addressed in: M. Bright E. Michielssen, J. Kusuma, “Gabor Frame-Based Sparsification and Radiation Boundary Conditions for Parabolic Wave Equations,” Propagation and Remote Sensing in Complex and Random Media. Thursday 16:00 – 17:40

Integral Equation: Weaknesses

A. Memory consumption / computational complexity
   - Addressed by fast methods such as MLFMA, SDFMM/spectrally accelerated methods, Butterfly, Rank-revealing schemes…

B. Atmosphere modeling
   - Use of VIEs is out of the question



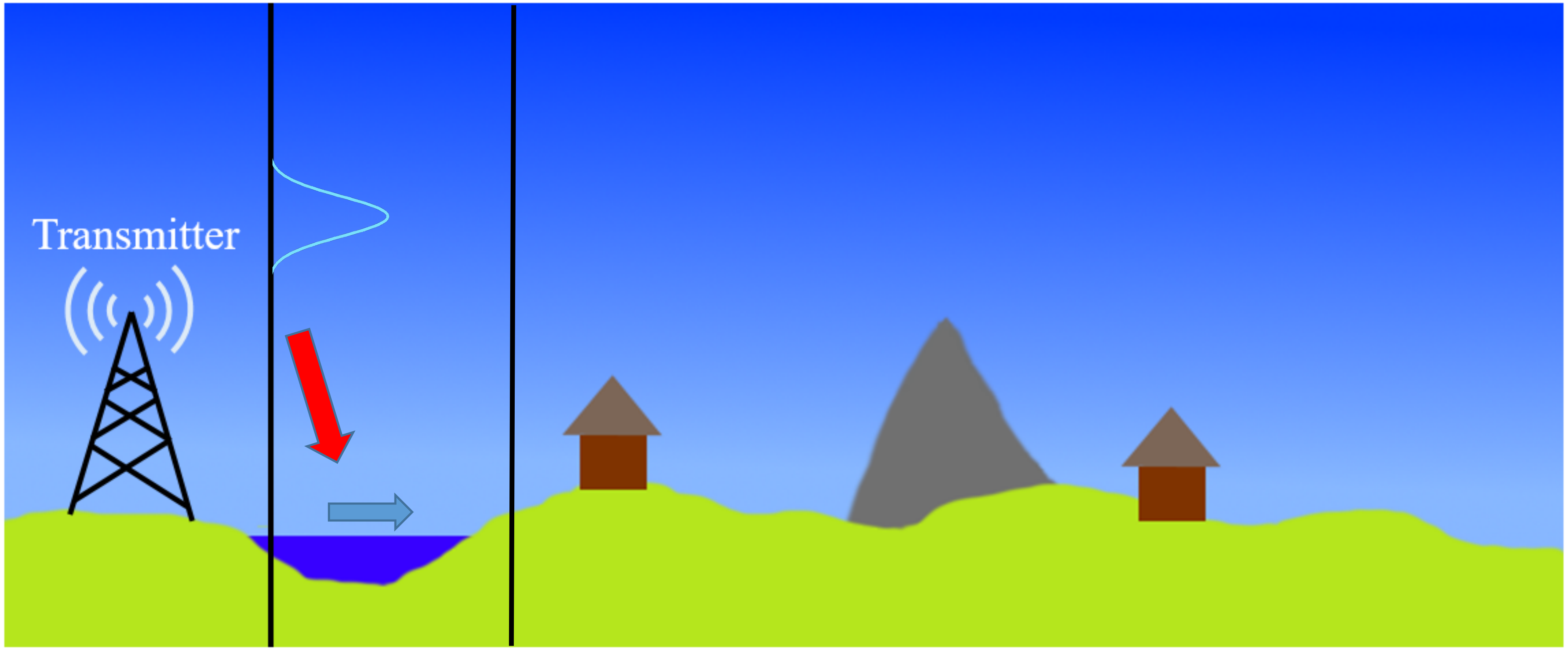
This contribution: Hybridization of Fast IE solver and PWE solver

- Noteworthy past effort at hybridizing PWE and IE: Rino, Charles L., and Hoc D. Ngo. "Forward propagation in a half-space with an irregular boundary." *IEEE Transactions on Antennas and Propagation* 45.9 (1997): 1340-1347.

Proposed Solver: Key Characteristics

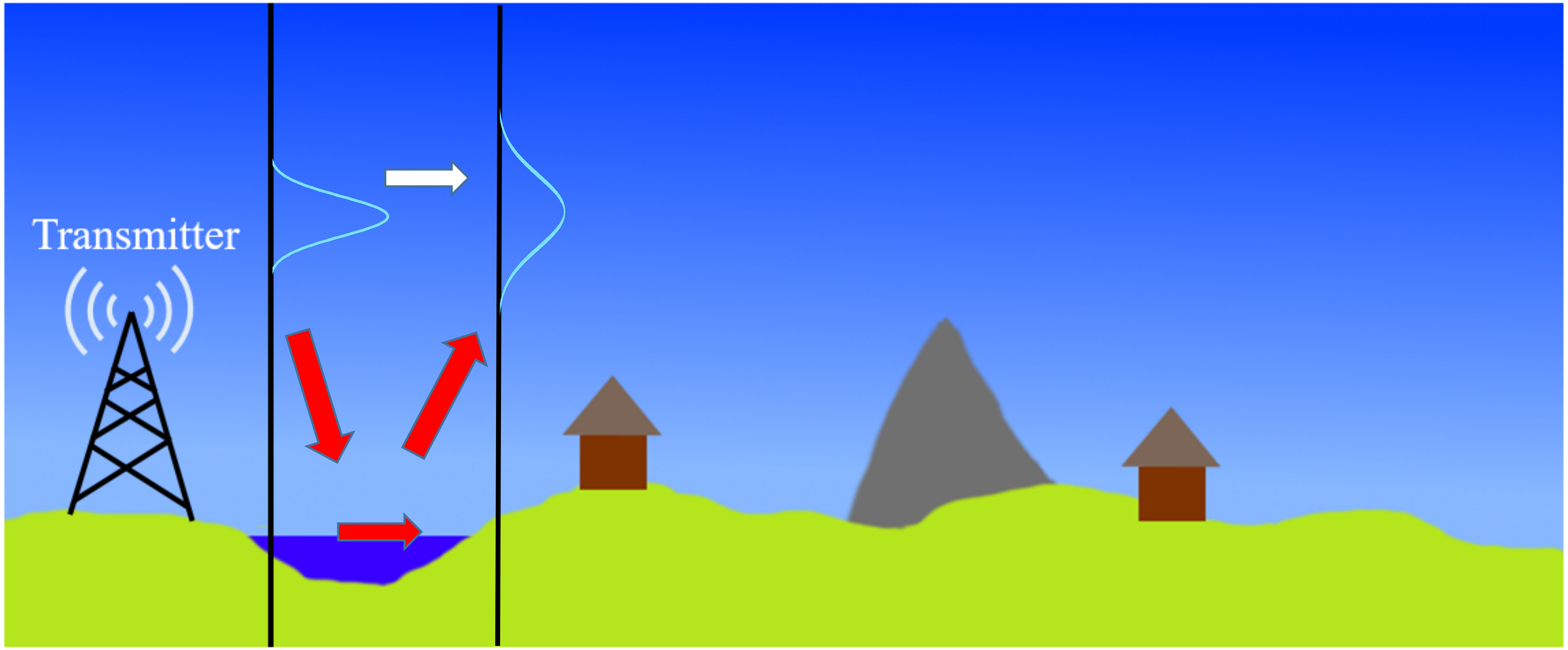
- **Integral Equation (IE) for surface**
  - IBC or dielectric interface
- **Green's function = background medium** (inhomogeneous atmosphere)
  - Computed using split step "spectrally resolved" PWE method
- **Discretized IE solved iteratively using forward-backward (FB) method**
  - Special case I: One forward sweep: traditional PWE
  - Special case II: Multiple sweeps in homogeneous medium : traditional FB / Gauss Seidel method
- **"Fast aspects"**
  - Low-rank scheme for "near-field" interactions
  - Fast hierarchical & spectrally accurate method for computing "PWE fields" from sources and back
- **Options: matrix-free / Gabor accelerated**
- **Cost scales as (N_x N_z log(N_z)) with very small leading constant**

Proposed Solver: Forward Sweep



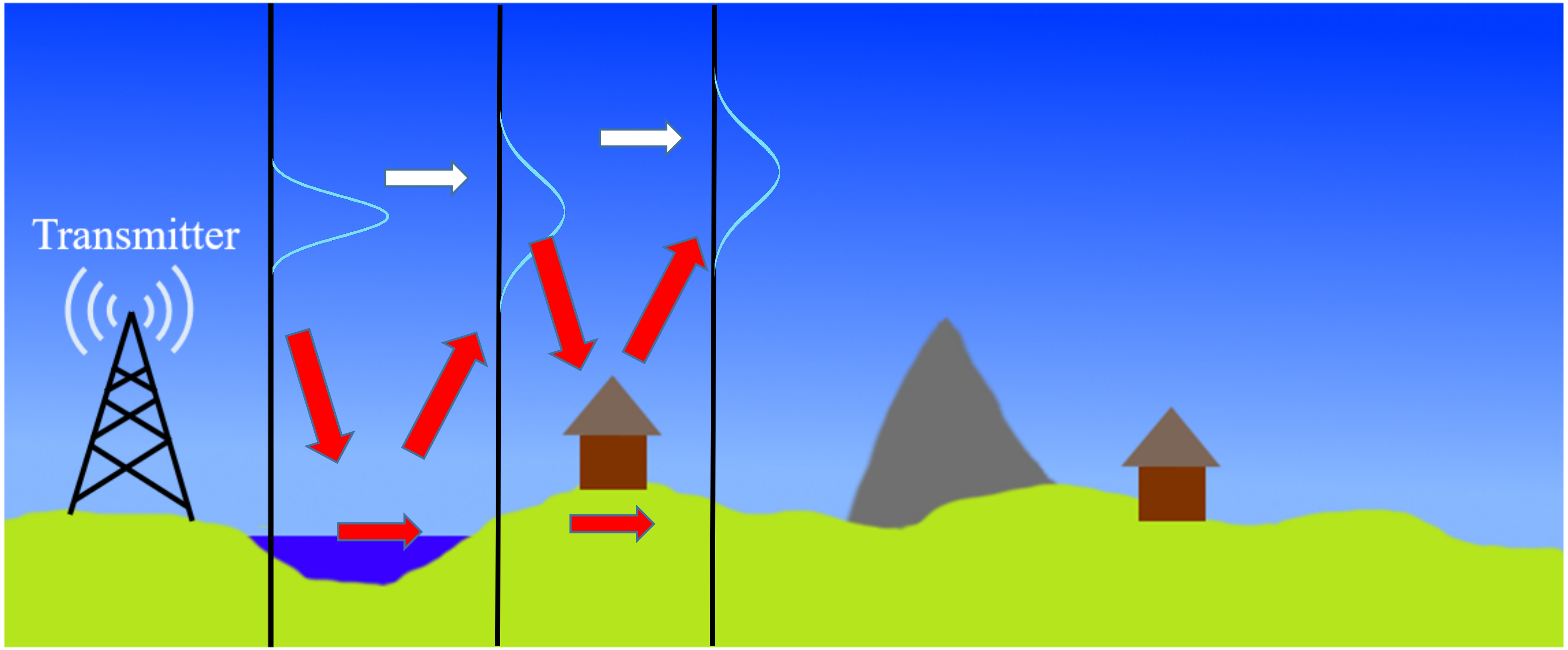
2. Compute "incident fields" on first surface slice. Use free-space Green function
3. Compute currents on surface = solve IE

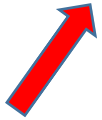
Proposed Solver : Forward Sweep



2. Compute “scattered field” on second PWE slice. Use free-space Green function
3. Add fields propagated by PWE

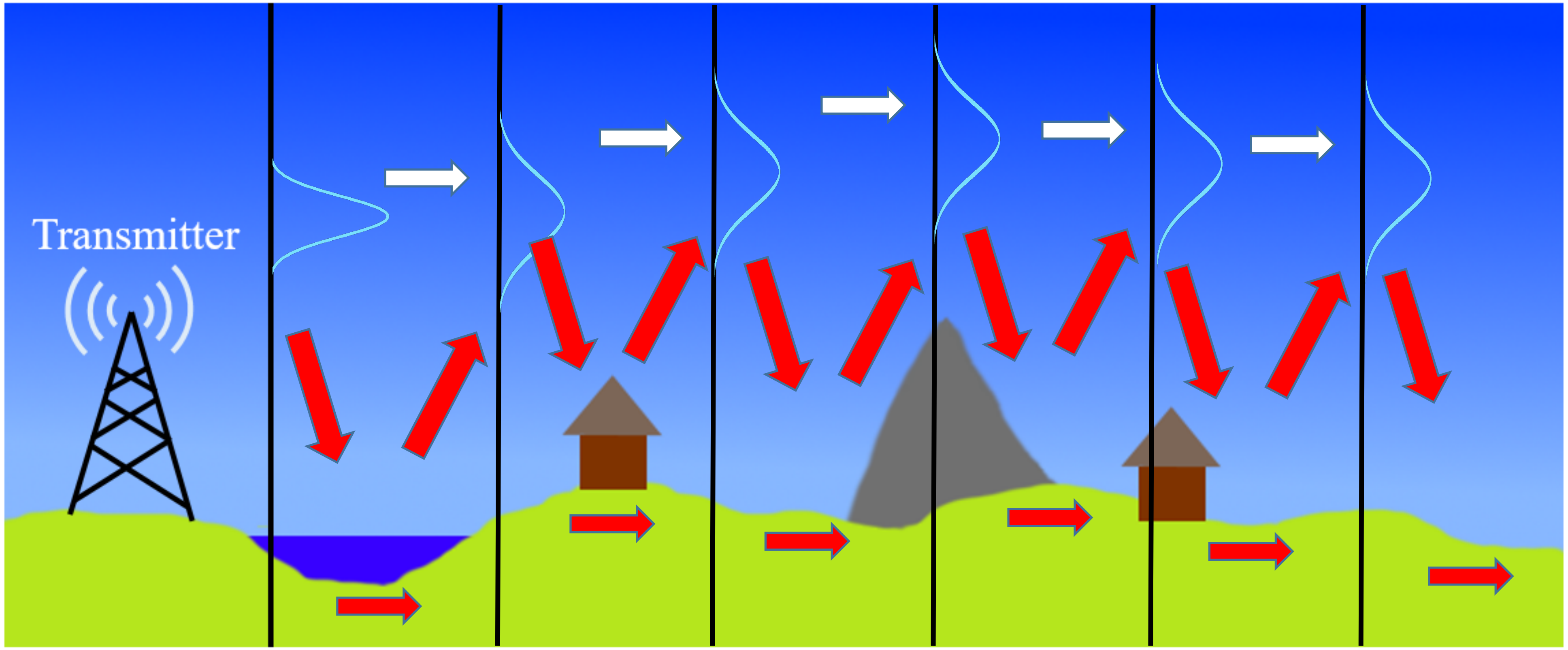
Proposed Solver : Forward Sweep



2. Compute “scattered field” on second PWE slice. Use free-space Green function 

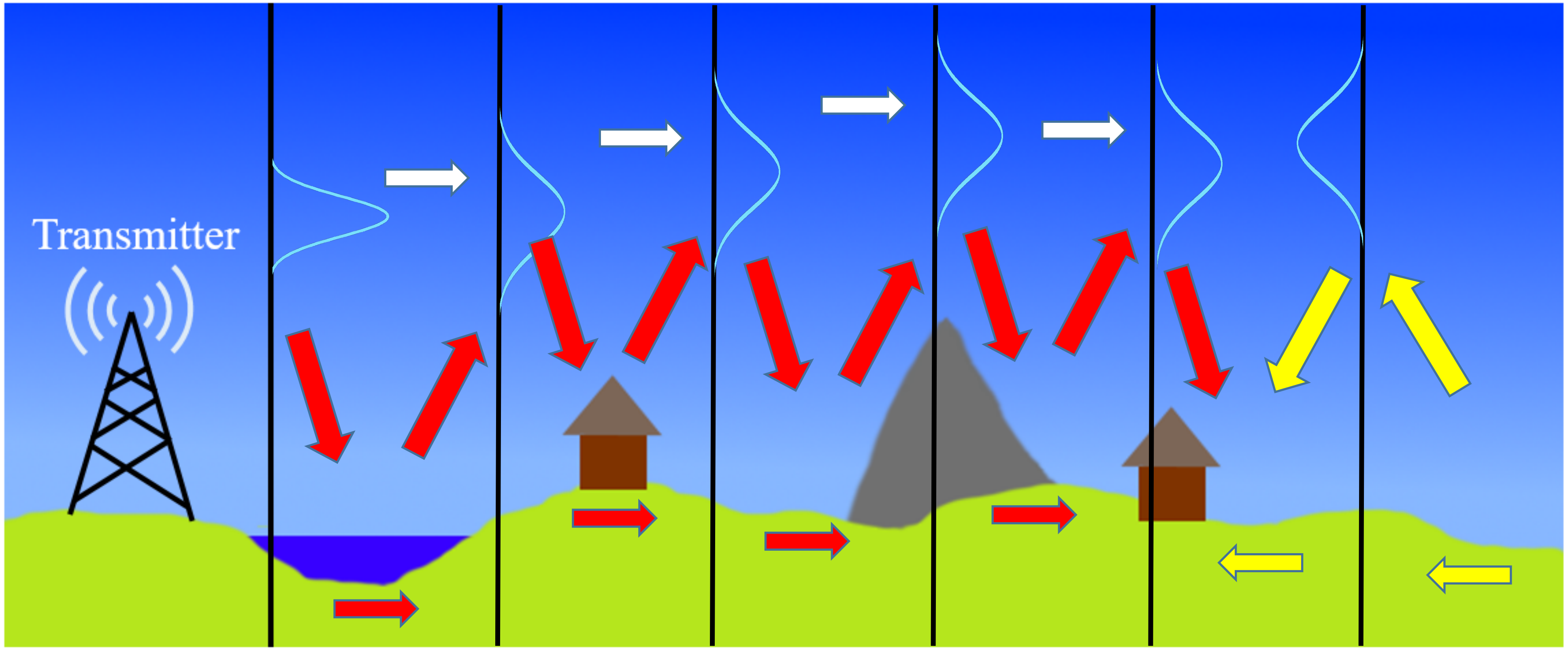
3. Add fields propagated by PWE 

Proposed Solver: Forward Sweep



Repeat for all slices: (i) compute incident fields on IE surface – (ii) compute currents – (iii) compute fieds on PWE slices – (iv) propagate and add PWE fields .

Proposed Solver: Backward Sweep



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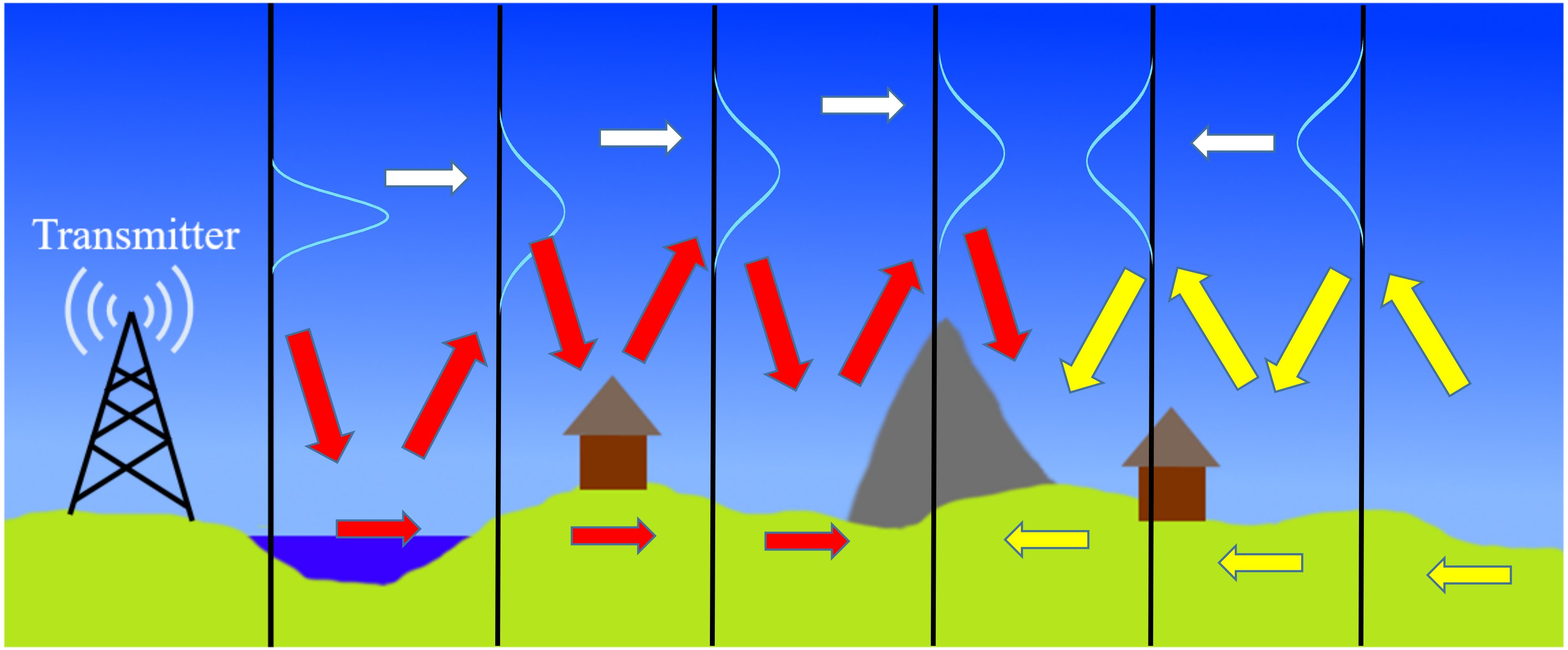

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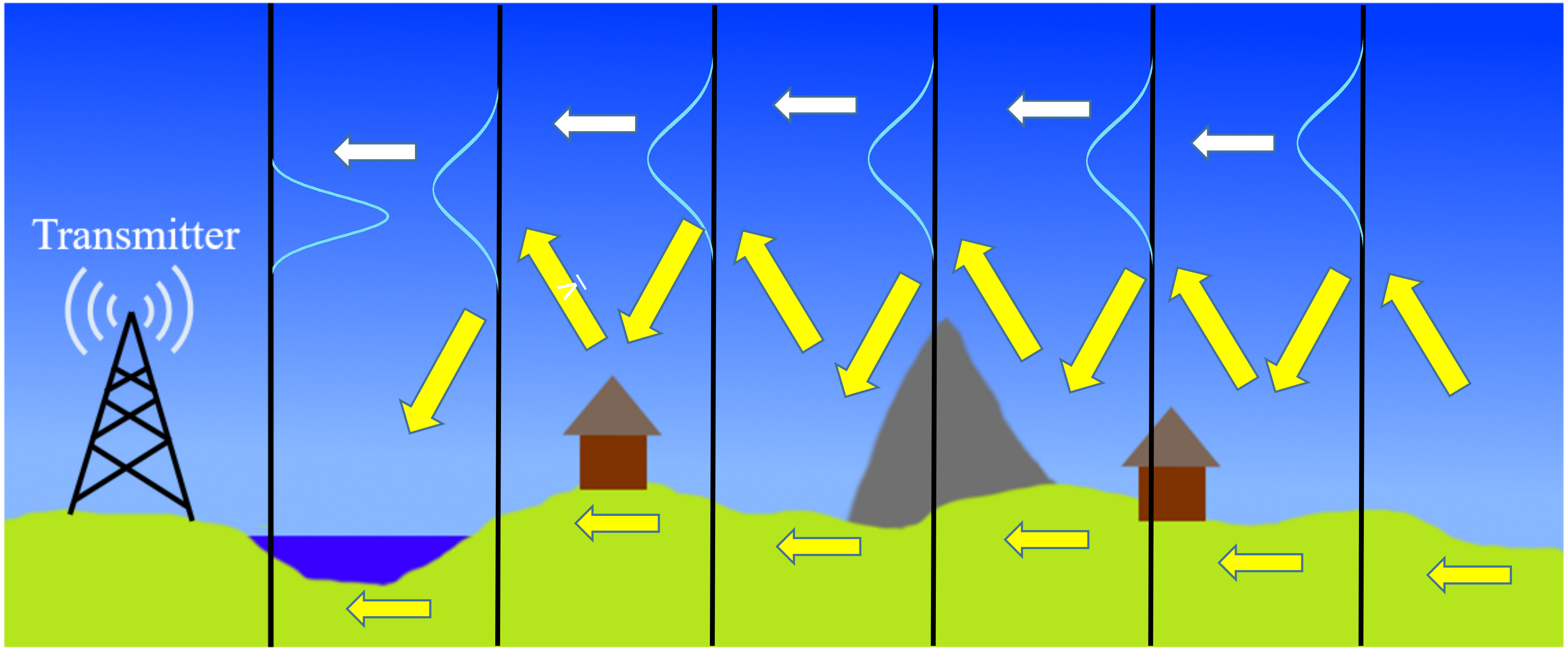
Proposed Solver: Backward Sweep



Repeat for all slices: (i) compute incident fields on IE surface – (ii) compute currents – (iii) compute fieds on PWE slices – (iv) propagate and add PWE fields .

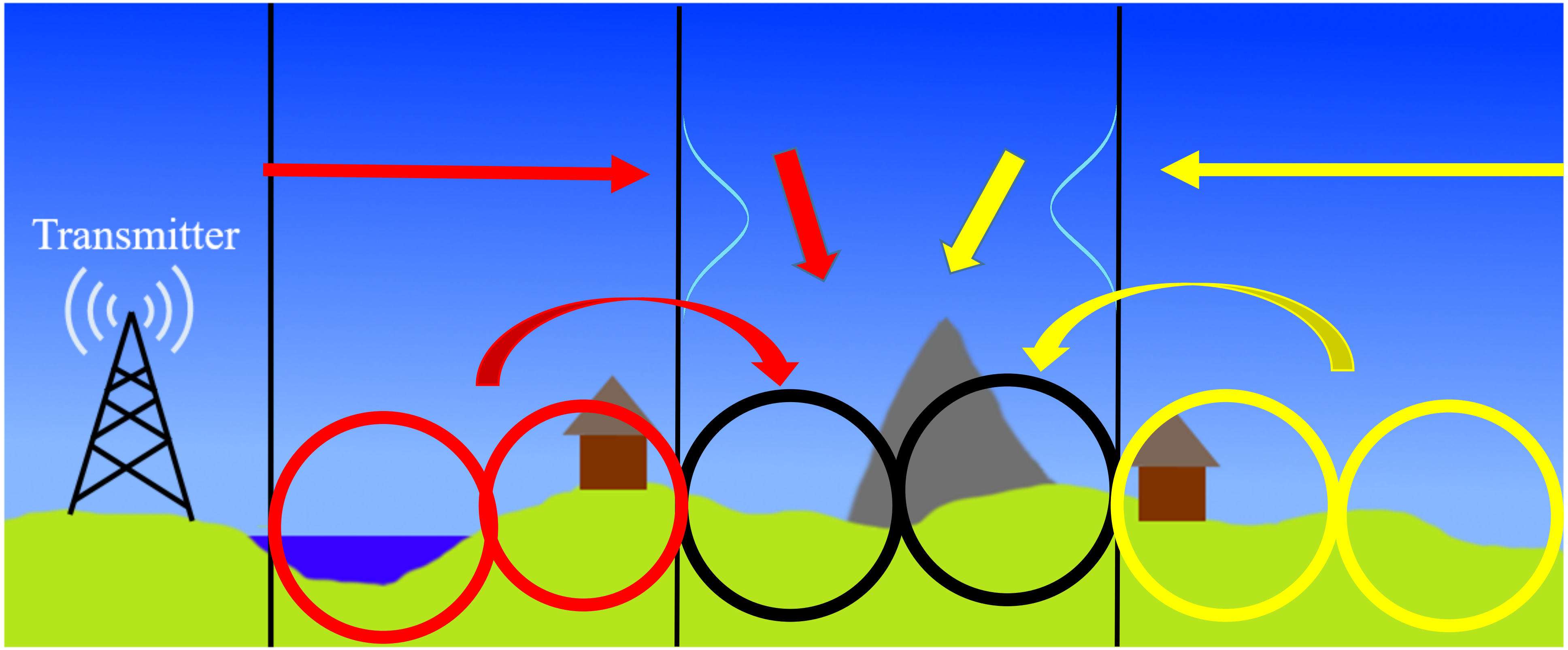
Proposed Solver: Implementation Details

- **Near-field buffer zone:**
  - **In practice "near-fields" (self *and* neighbor interactions) are computed all using IE**
  - **Fields from sources are only added to PWE fields beyond buffer region**
  - **PWE discretization does not have to account for evanescent fields – same sparse discretization as for classical PWE solvers can be used**

- **Any IE solver and acceleration method can be used**

- **Computation of PWE fields produced by sources**
  - **Achieved using windowed Weyl transform**
  - **Real spectrum, compatible w/ PWE**
  - **Fast interpolation/filtering schemes to transition from IE to PWE spectral requirements**

Proposed Solver: Implementation Details I

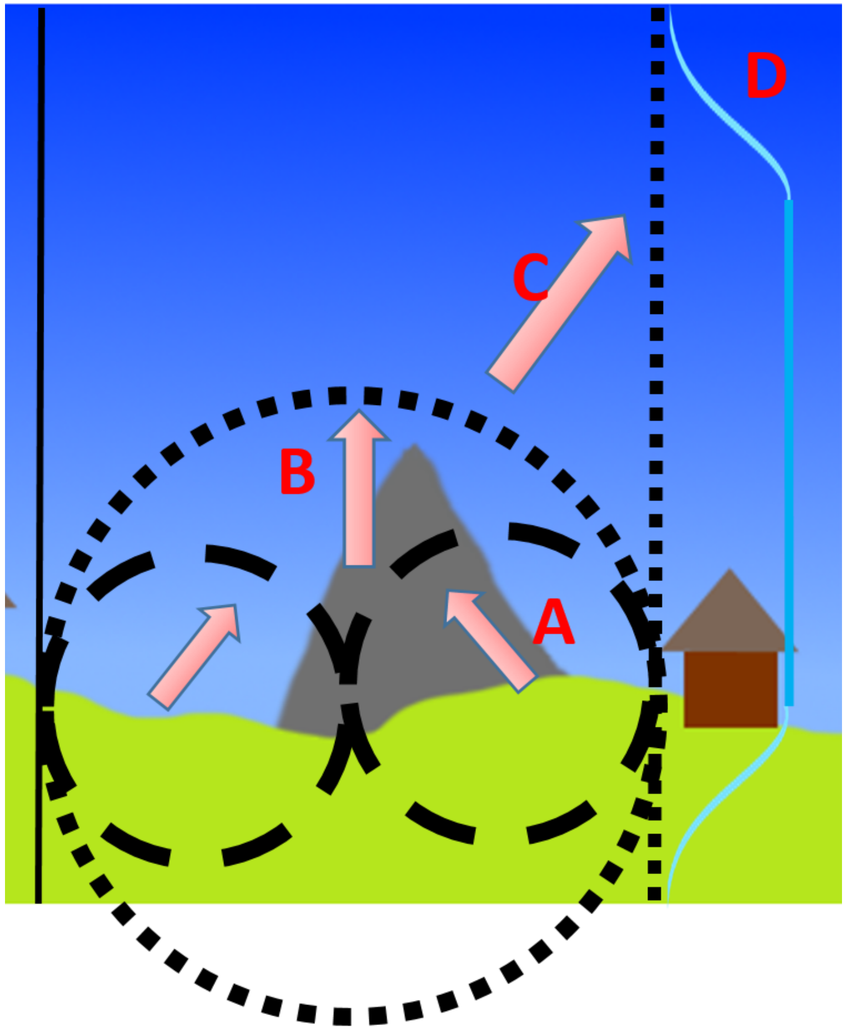


Near-field buffer zone:

In practice “near-fields” (self and neighbor interactions) are computed all using IE

Fields from sources are only added to PWE fields beyond buffer region FMM-style

Proposed Solver: Implementation Details II



A : form cylindrical/spherical FMM plane wave spectra at lower levels

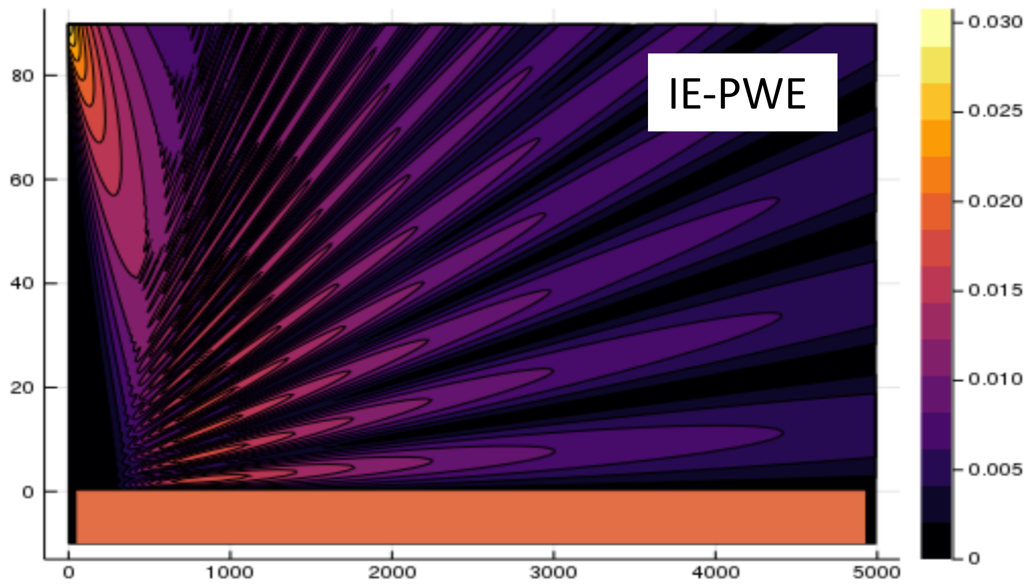
B : form cylindrical/spherical FMM plane wave spectra at higher levels

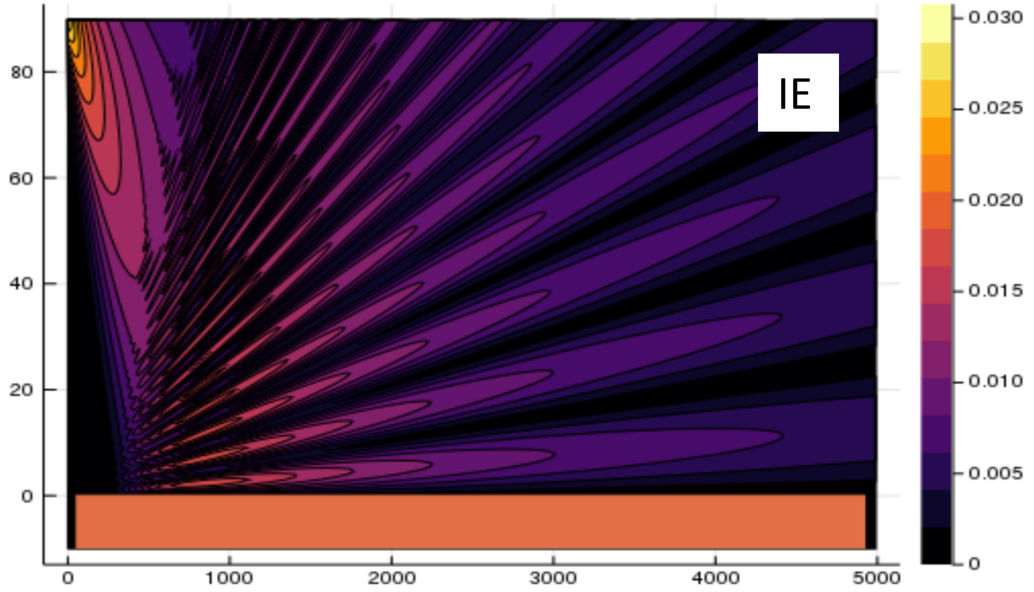
C : convert cylindrical/spherical FMM spectra to planar plane wave spectra accounting for Weyl weight and spectral window

$$H_0^{(2)}(k\rho) \approx \frac{1}{\pi} \int_{-k_0}^{k_0} \frac{e^{-jk_x x - k_z z}}{k_x} W_{Spectral}(k_z)\, dk_z$$

D : Apply spatial window to capture rays escaping domain

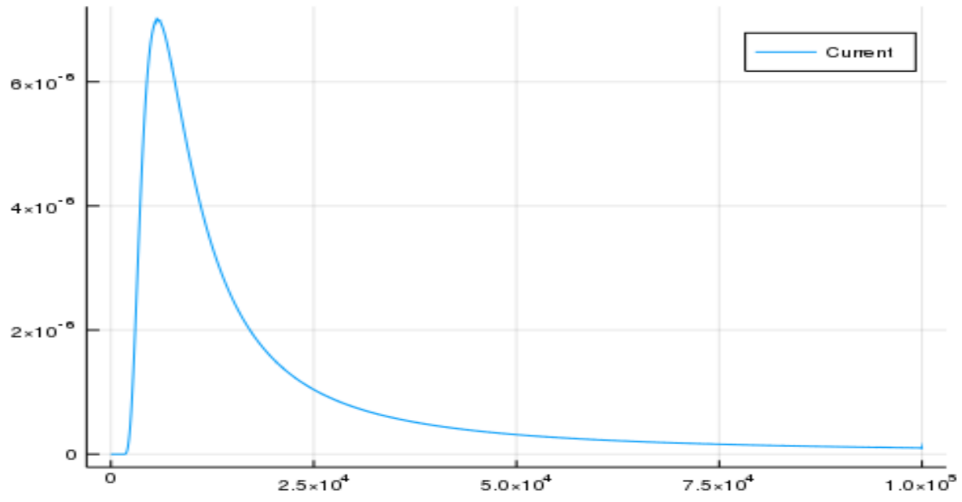
Numerical Result 1: Scattering from Flat PEC Ground





H-pol – 300 Mhz
Range = 5 km
N = 100,000

Current on plane



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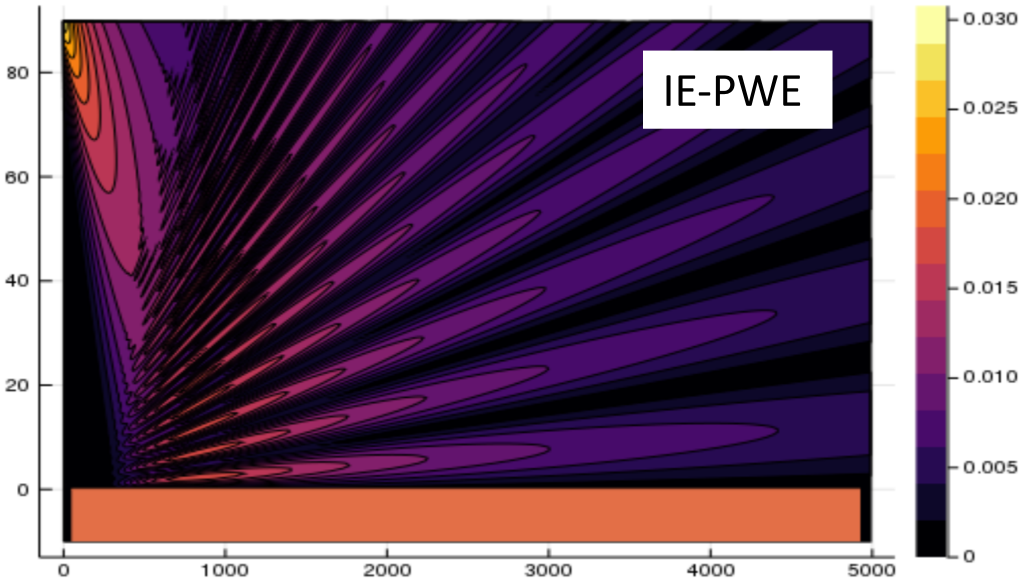
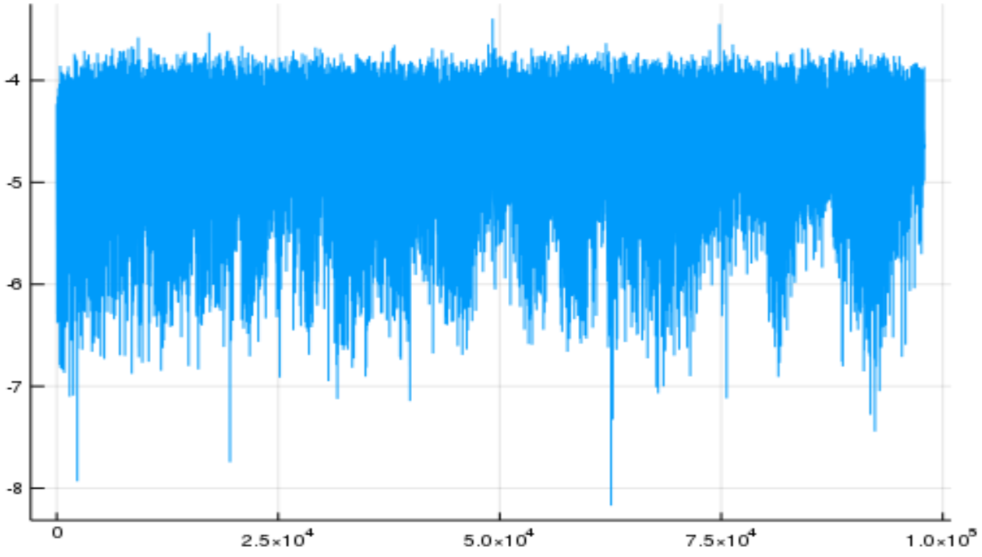
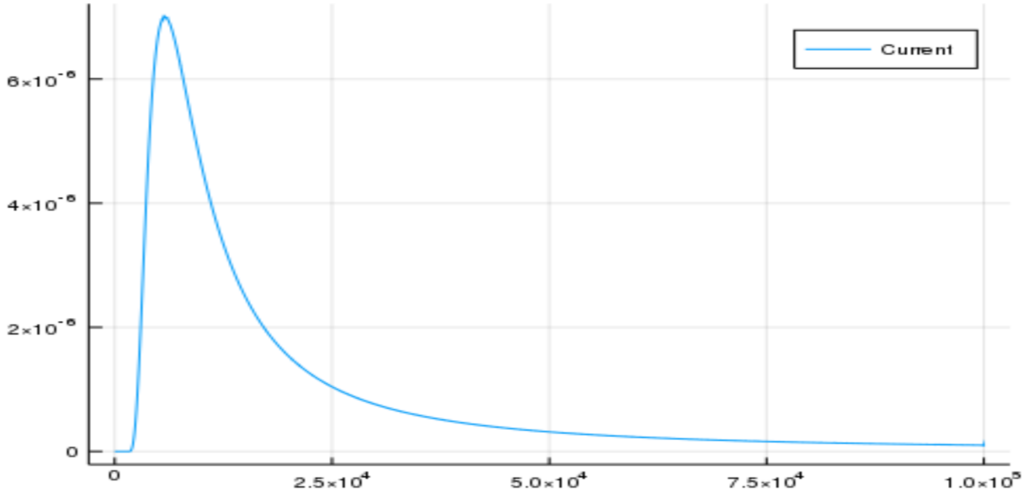


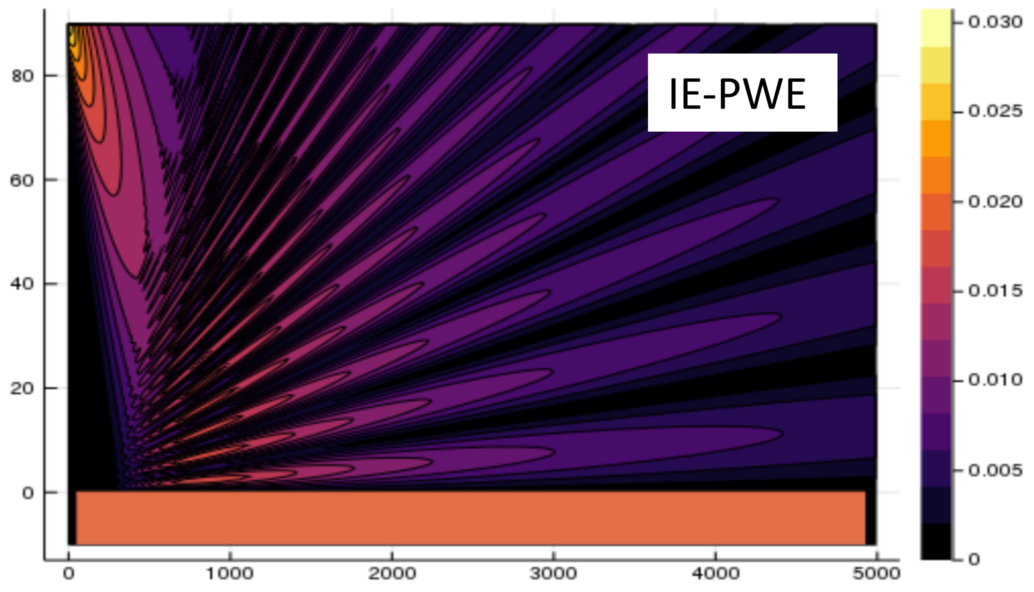


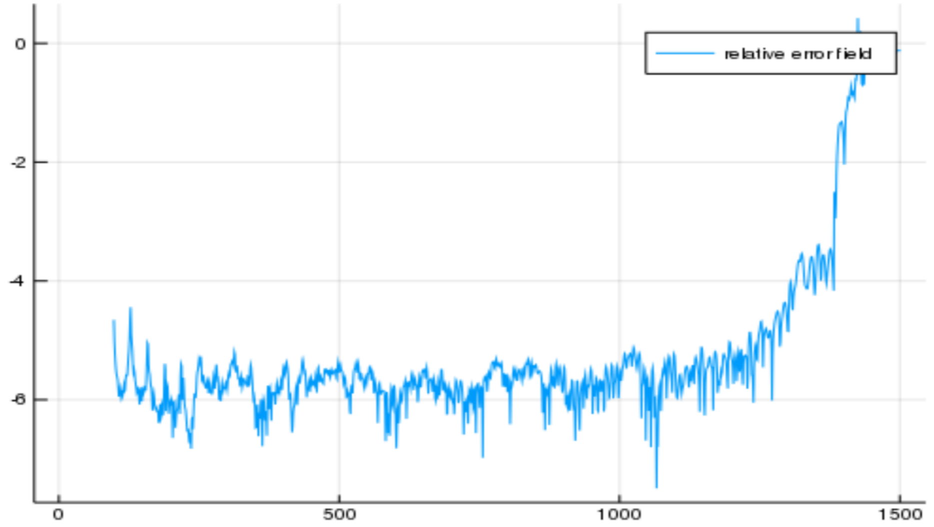
H-pol – 300 Mhz
Range = 5 km
N = 100,000

Currents on surface
Current Accuracy: ~ 4 digits



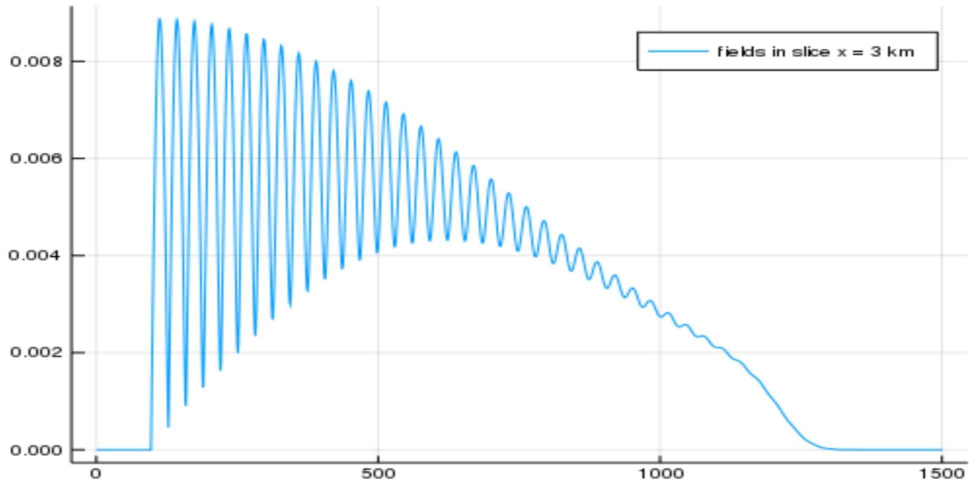
Numerical Result 1: Scattering from Flat PEC Ground



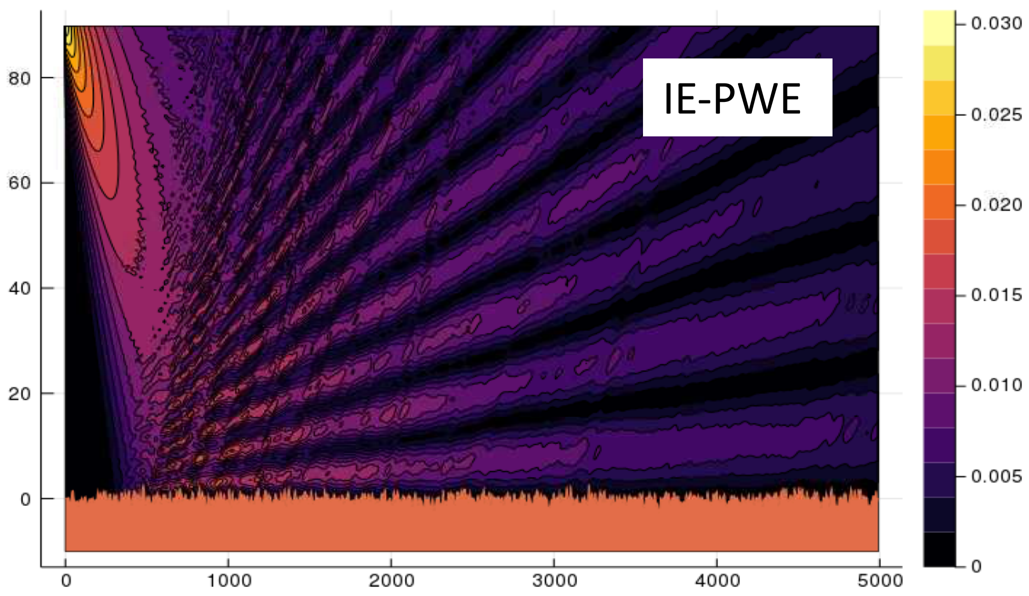


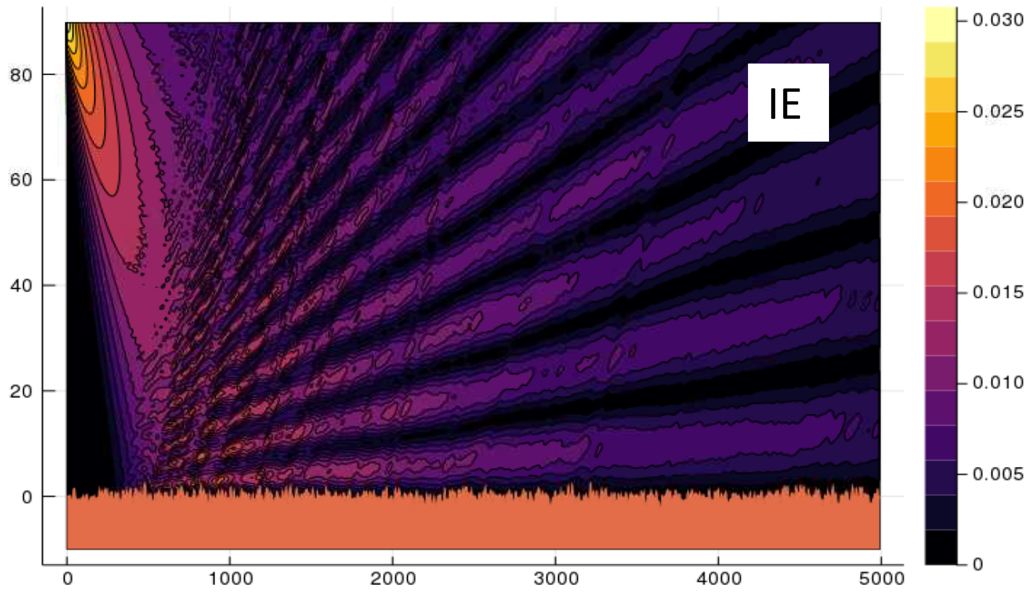
H-pol – 300 Mhz
Range = 5 km
N = 100,000

Fields in slice at x = 3 km
Accuracy = 4+ digits

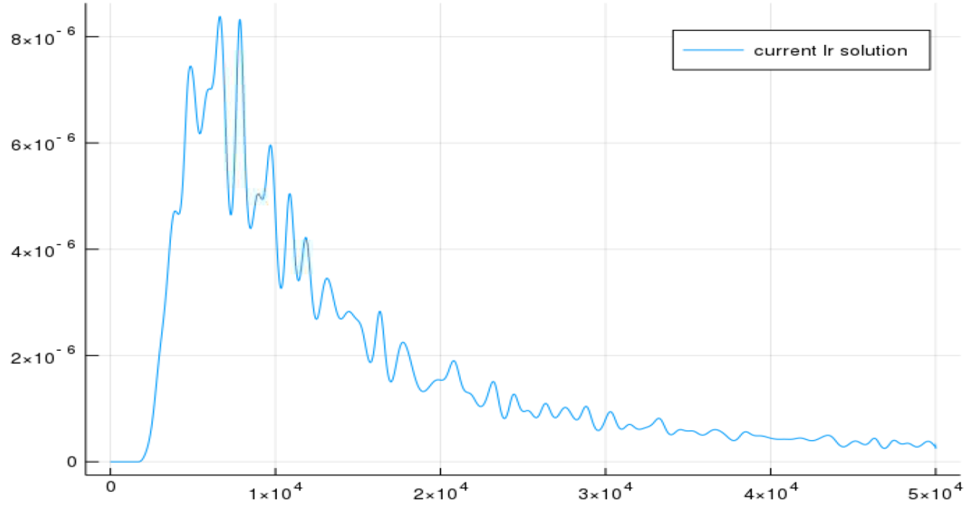


Numerical Result 2: Scattering from Rough Surface





RS – Corr. Length = 3m;
RMS height = 1m – dry soil
V-pol – 300 Mhz
Range = 5 km
N = 100,000
Current on plane



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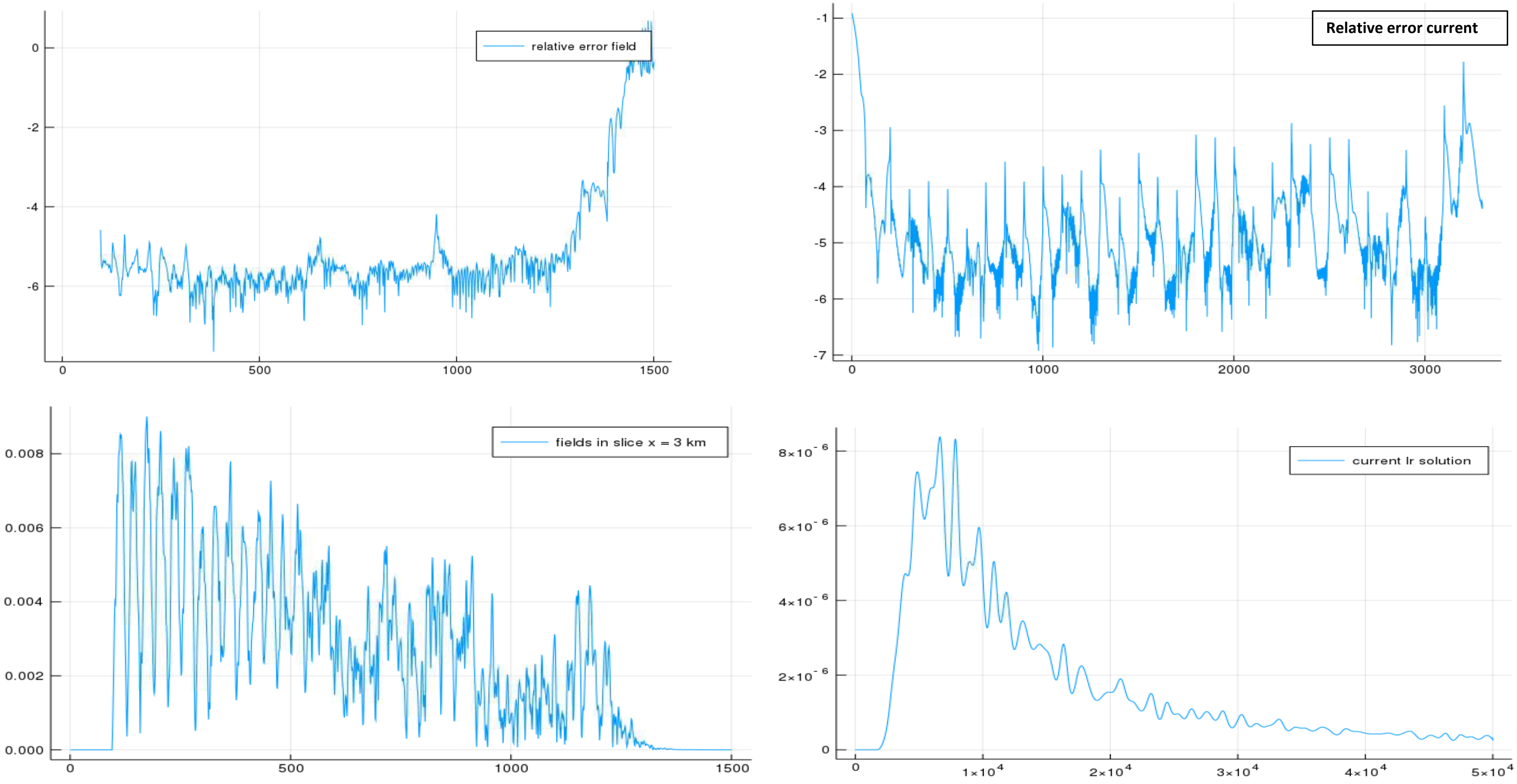
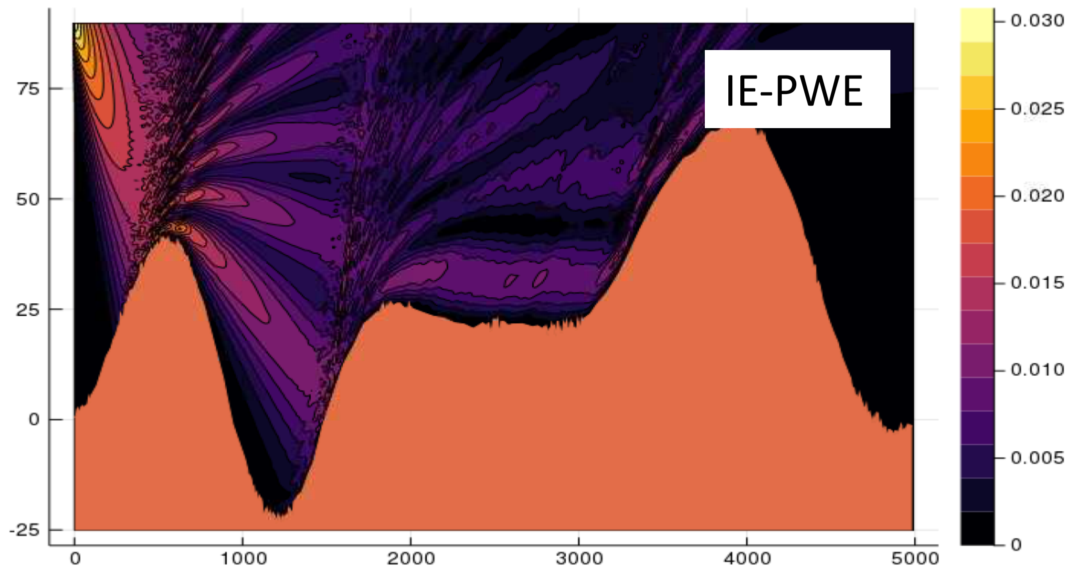
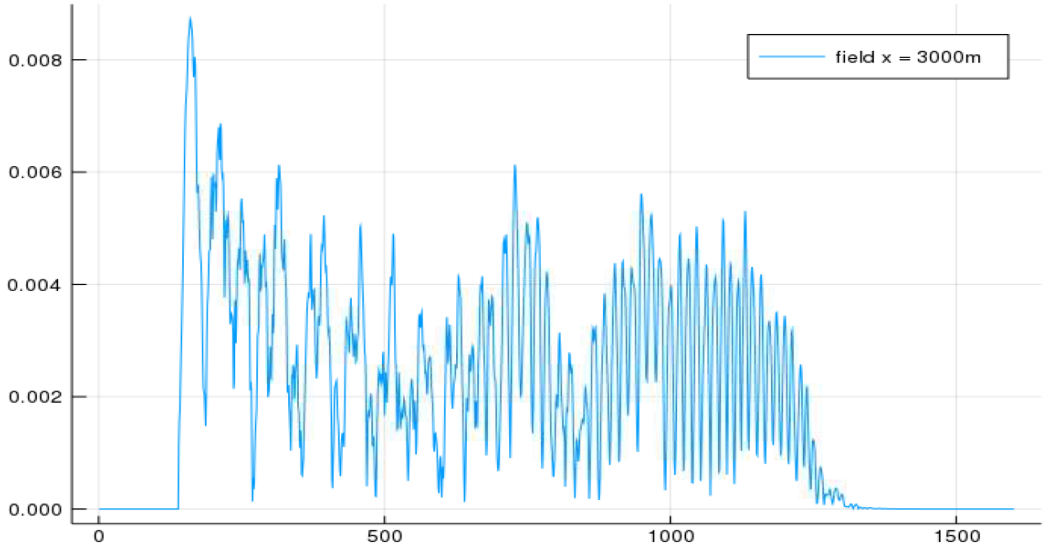
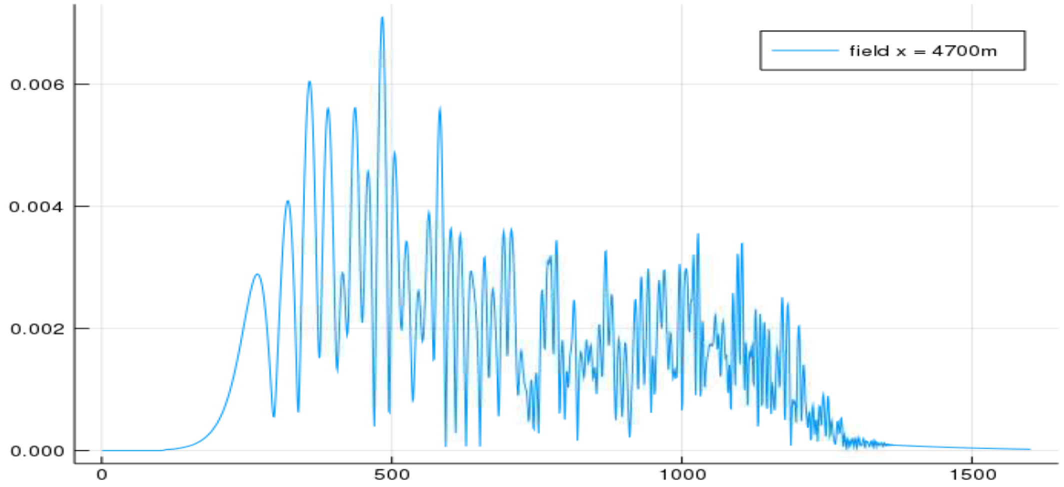


Numerical Result 3: Scattering from Hilly Terrain w/ Rough Surface

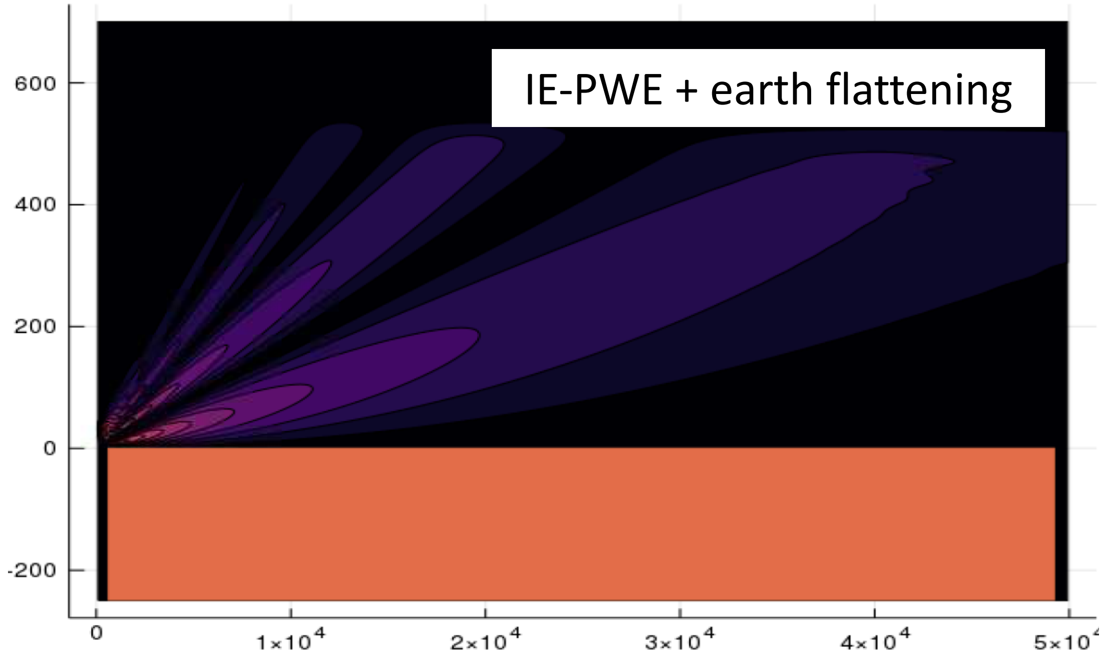


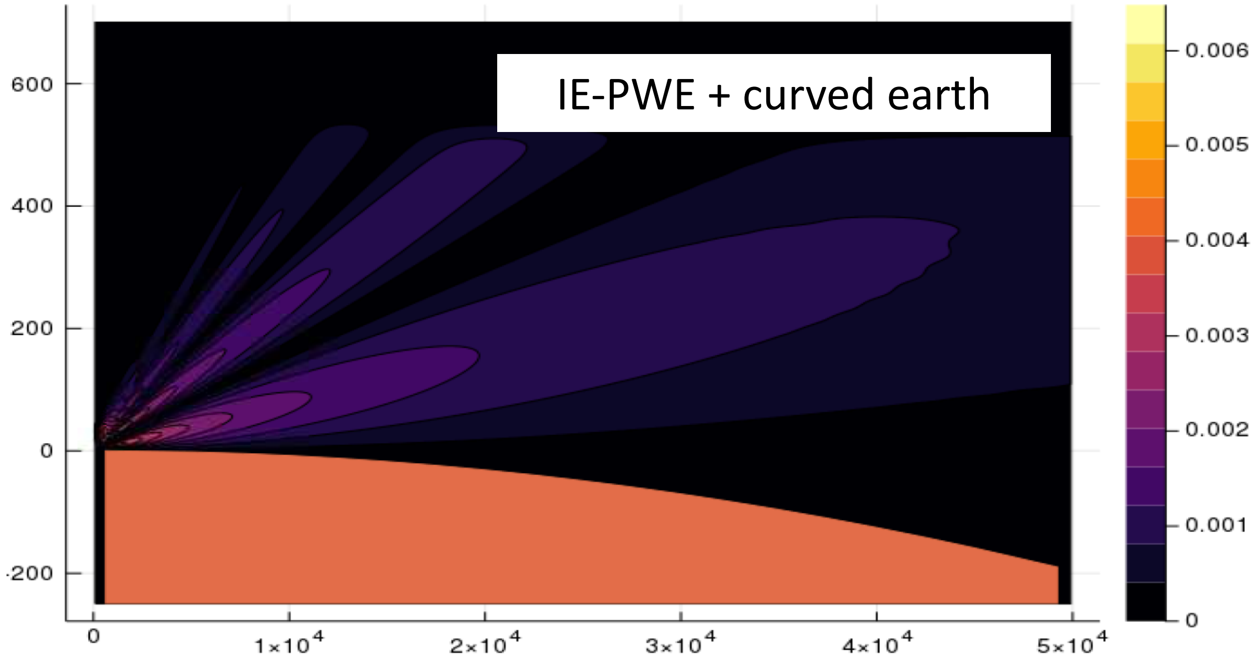


RS – Corr. Length = 3m;
RMS height = 1m – dry soil
V-pol – 300 Mhz
Range = 5 km
N = 100,000
Fields in two vertical slices



Numerical Result 4: Antenna on Curved Earth





IE-PWE solver with "earth flattening atmosphere": n = 1+z/a

IE-PWE solver with explicitly curved earth: n = 1

H-pol – 300 MHz – antenna h = 50m

Results match : beam tilts are within a few meters after 50km

Numerical Result 5: Exaggerated Duct Over Sea

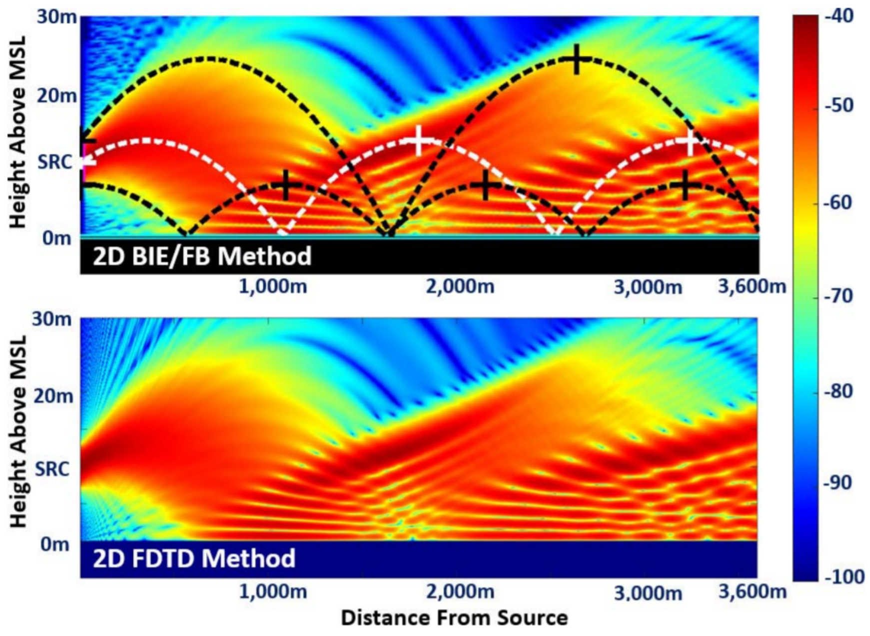
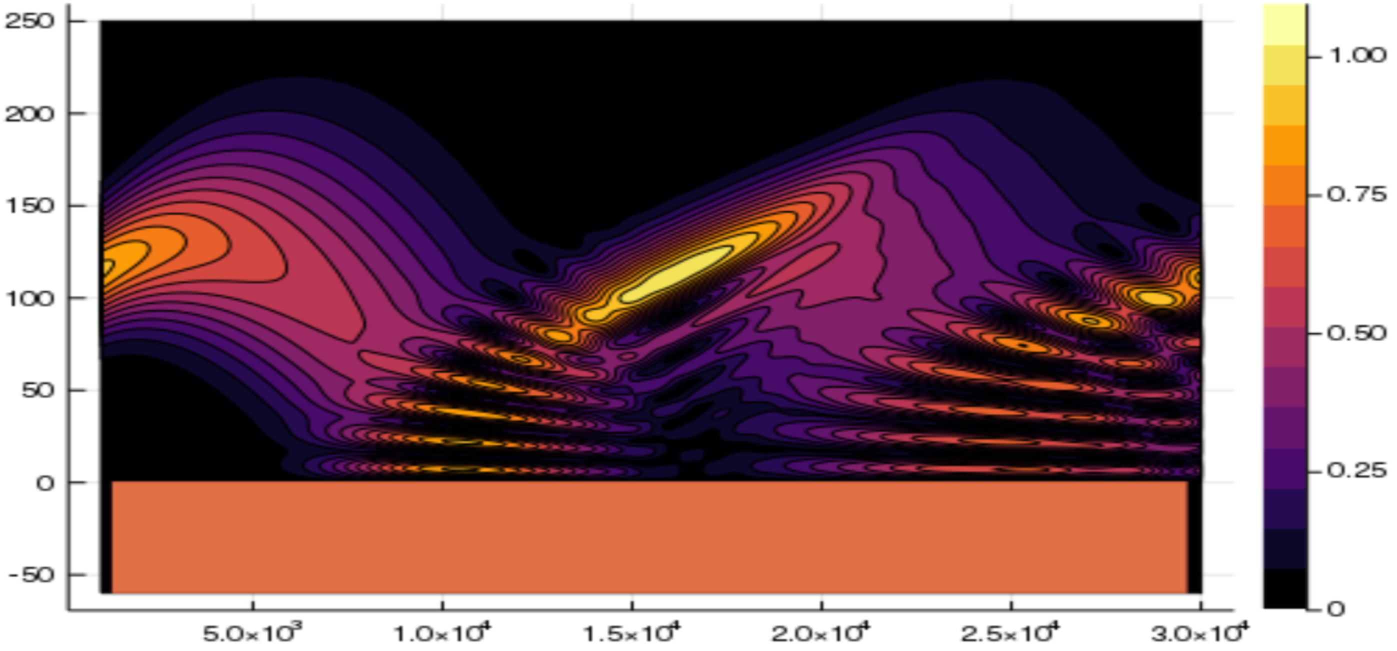


Fig. 13. Total field of a high directivity 3 GHz source in the presence of an exaggerated duct. BIE result (top) [8] (by permission). FDTD (bottom).



C. Bourlier, H. Li, and V. Fabbro, "Radar propagation modeling using the boundary integral equations in a maritime environment with a duct," in Proc. Int. Radar Conf., Oct. 2014, pp. 1–5

Brandon W. Dowd and Rodolfo E. Diaz. "FDTD simulation of very large domains applied to radar propagation over the ocean." *IEEE Transactions on Antennas and Propagation* 66, no. 10 (2018): 5333-5348

Flat sea – eps = 70.4 – j 40.6

H-pol – 3 Ghz

Range = 3.6 km

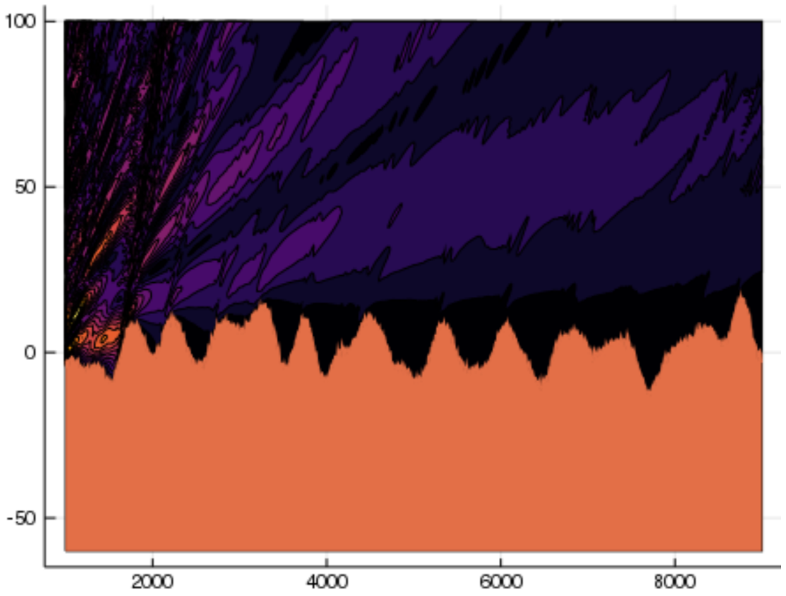
N ~ 90,000

n = Sqrt(1 + 0.0001 (50 – z))

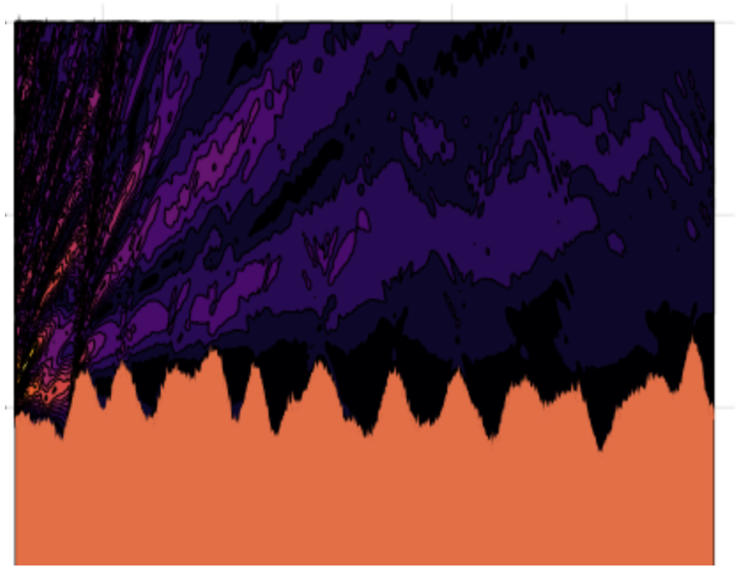
Numerical Result 5:
Convergence of Forward Backward Iterative Solver

- “RS” – Corr. Length = 300m;
  RMS height = 10m – dry soil
- V-pol – 3 GHz – weak evaporation duct
- Range = 10 km - N = 2,000,000

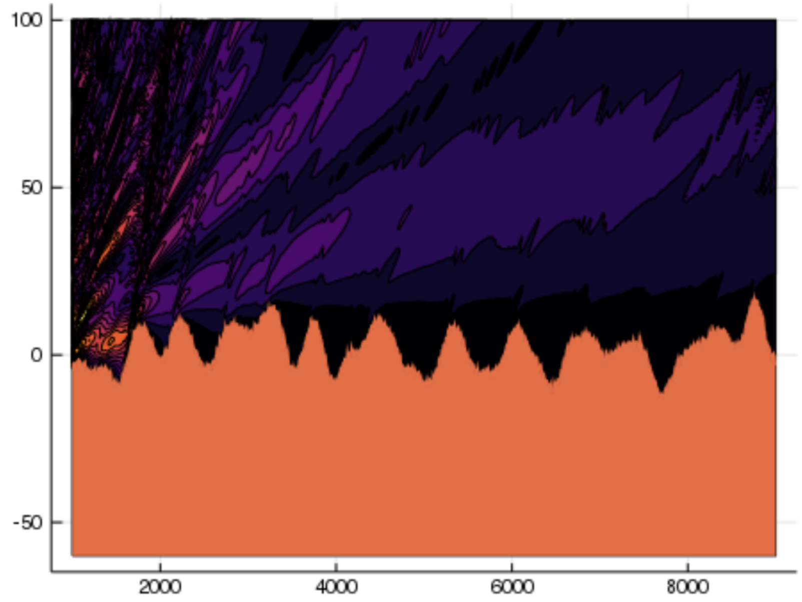
- Solver converges to err = 10^(-6) in 15 iterations
- Relative accuracy after 3 and 1 iteration are 15% and 3%



Iteration = 15

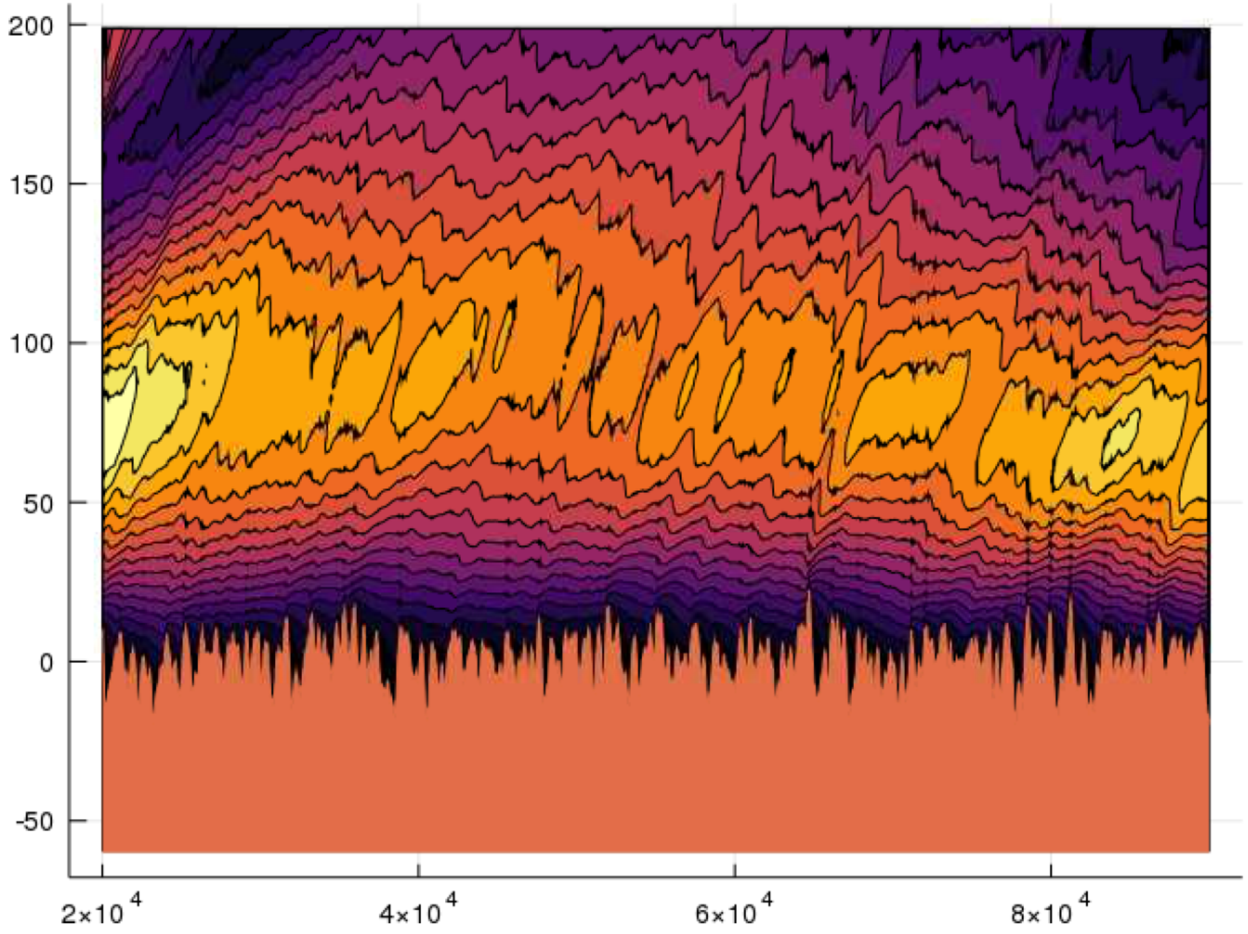


Iteration = 3



Iteration = 1

Numerical Result 6:
Convergence of Forward Backward Iterative Solver



"RS" – Corr. Length = 100m;
RMS height = 10m – dry soil

H-pol – 3 Ghz

Low duct

Range = 100 km

N ~ 10,000,000

Nz = 8192

Solution time ~ 2 hrs

Conclusions

- New IE – "PWE" hybrid
  - Uses (new) plane wave translation scheme
  - sole angle restricting operation is split step scheme)
- Applies to large domains, complex terrain, and realistic atmospheric profiles
- Computational expense = O(Nx Nz log(Nz)) with very small leading constant.
- No dispersion error
- Can be hybridized with Gabor/ray based propagator to sparsify field representations
- 3D is within reach