Z-Qualities and Renderable Mass-Damping-Stiffness Spaces: Describing the Set of Renderable Dynamics of Kinesthetic Haptic Displays

Nick Colonnese[1] and Sonny Chan[2]

***Abstract*— In this paper we define language and definitions to define the renderable set of dynamics that a general kinesthetic haptic display can render to a human operator. This is accomplished in three steps. First, we present a model that applies to every kinesthetic haptic display. Then, we define the *Z-Qualities* of a haptic display: characteristics that describe the display's stability, sensitivity to instrumentation error, speed of changing the rendered dynamics, and accuracy of the rendered and desired dynamics. Finally, we define the *Renderable Mass-Damping-Stiffness Spaces* of a haptic display: the set of mass-damper-spring impedances that the display can render that satisfy specified Z-Quality constraints. We highlight existing key results for various Z-Qualities, and provide illustrative examples of renderable mass-damping-stiffness spaces for popular specified Z-Qualities 'Passiva' and 'Stabila.' This work aims to provide a framework for determining if a given haptic display can render dynamics with certain qualities, and we hope is particularly useful for psychophysical and scientific studies where accurate rendered dynamics to the human are essential.**

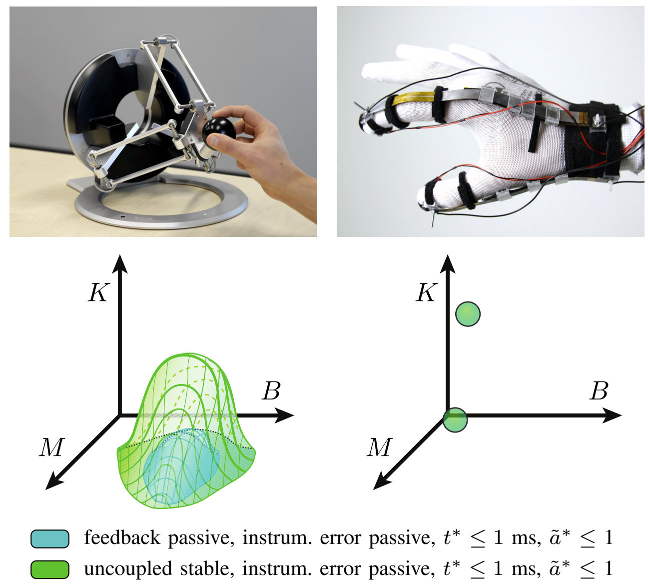


Fig. 1. Kinesthetic haptic displays (the Force Dimension omega.3, left, and the DextrES [1], right), and illustrative representations of their respective Mass-Damping-Stiffness Spaces for two specified Z-Qualities. The shaded volumes show the set of mass-damper-spring impedances that are renderable by the displays that satisfy specified stability, instrumentation error, speed, and accuracy, conditions.

I. INTRODUCTION

Kinsesthetic haptic displays render forces to a human operator corresponding to a virtual or remote environment. They are used in many fields such as surgery, mining, rehabilitation, and most recently, virtual and augmented reality.

The capability and quality of a kinesthetic haptic display is described by the set of *dynamics*, a relationship between motion and force, that it can render to a human operator. Dynamics can be expressed in several ways: a differential equation, a Bode plot, or most typically, a rational transfer function representing either the output force to input velocity, impedance $Z(s)$, or output velocity to input force, admittance $Y(s)$. Simple examples of dynamics are that of a spring, (force proportional to position), a mass (force proportional to acceleration), and free space (no force from any motion). A complex example of dynamics is the force and motion relationship of an impact hammer hitting an aircraft wing.

Describing the set of renderable dynamics of a haptic display is fundamental for many haptic scenarios in which precise characterization of the mechanical properties rendered to the human are essential. For example, consider a psychophysical experiment that requires rendering dynamics of two springs with different stiffnesses to the human. The validity of the experiment requires that for each spring stiffnesses rendered to the human the display must be a) robustly stable to various grasps of the human b) free of haptic impulses or vibrations from instrumentation error, c) render the correct spring stiffness sufficiently quickly, and, most importantly, d) render the correct spring stiffness within certain accuracy to the desired stiffness. In this scenario, and others with exacting specifications for what is rendered to the human, the haptic practitioner desires an answer to the seemingly simple question 'what set of dynamics can a given haptic display render?' Unfortunately, answering this question is challenging for several reasons.

One challenge is that the dynamics rendered to a human are a function of all of the components of a haptic display: the mechanical design of the haptic device, the computer-generated virtual environment, and, the control approach mediating forces between the user and virtual environment. Usually these components are created/designed in isolation, and a holistic description of the complete haptic display is not readily available or straightforward to generate.

Another challenge is that defining if a haptic display can faithfully render certain dynamics depends on what *qualities* (e.g., stability or sensitivity to instrumentation error properties), of the rendered dynamics are required. Different haptic applications call for different qualities, and thus, a widely-accepted concept of 'renderability' is not well-

[1]Facebook Reality Labs, 9845 Willows Road, Redmond, WA, 98052 nick.colonnese@fb.com
[2]Department of Computer Science, The University of Calgary, 2500 University Drive N.W., Calgary, Alberta, T2N 1N4, Canada. sonny.chan@ucalgary.ca

defined. For example, in one application it may be sufficient for the haptic display to be uncoupled stable and have small intermittent instrumentation error impulses, while in another application, the haptic display must be unable to produce any energy from both the structure of its haptic feedback loop and instrumentation error. Without first providing definitions for the required qualities of the rendered dynamics, it is impossible to answer what sets of dynamics a haptic display can faithfully render.

II. Related Work and Contributions

Haptic displays researchers have predominately used two different approaches to describe the set of renderable dynamics of a kinesthetic haptic displays.

One approach uses the concept of *Z-Width*, which is (using language defined in this paper for clarity), 'the set of feedback passive dynamics a haptic display can render to a human operator' [2]. Z-width remains the de facto approach today, but suffers from two important problems. First, Z-Width only speaks to a single *quality*: feedback passivity. It doesn't speak to other concepts of stability, or other important qualities such as instrumentation error, speed, or accuracy. Second, Z-Width is a *set* of dynamics, and thus, difficult to communicate. For this reason, it is common to use a one-dimensional 'quasi Z-Width' defined as 'the difference between the maximum and minimum magnitude of the rendered impedance as a function of frequency.' The quasi Z-width is simple to communicate, but loses most of its descriptive power: only the magnitude of the impedance difference is communicated, not what is being rendered.

Another approach is to assume that the dynamics rendered to the human are essentially that of the commanded control law. In this way, the contribution of 'parasitics' to the rendered dynamics such as the dynamics of the haptic device, time delay within the feedback loop, or low-pass filtering to achieve stability or instrumentation error objectives, are negligible. Using this approach, results are presented for the *commanded* control law parameters in contrast to the *rendered* dynamics. This approach can speak to qualities other than feedback passivity, such as uncoupled stability [3], [4], [5], [6], [7], instrumentation error [8], [9], [6], and speed [10], [7], but does not generally speak to the true, rendered, dynamics to the human. For popular haptic display architectures there exists analyses of the accuracy between the rendered and desired dynamics [11], [12], [13], but these results do not generalize to all kinesthetic displays. Table I displays the state of the research in various qualities of the dynamics rendered to the human.

The purpose of this paper is to provide a framework with which to rigorously define and communicate the set of renderable dynamics a kinesthetic haptic display can render to a human. First, in Section III we define terminology and introduce a model that applies to every kinesthetic haptic display regardless of mechanical design or control structure. Then, in Section IV, we define the *Z-Qualities* of a haptic display: characteristics that describe important properties of the rendered dynamics. Finally, in Section V, for use as a communication tool, we define the *Renderable Mass-Damping-Stiffness Spaces* of a haptic display: the set of all mass-damper-spring impedances that the display can render that satisfy specified Z-Quality constraints.

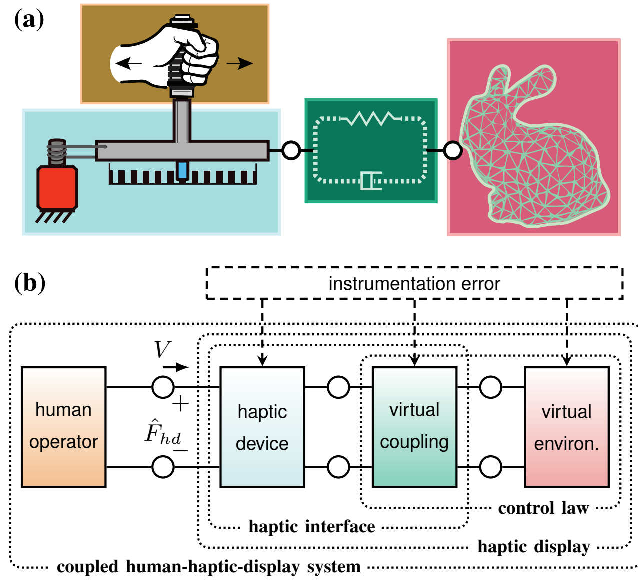


Fig. 2. Schematic, (a), and corresponding network representation, (b), of a haptic simulation. The dynamics rendered to the human are completely described by the *haptic display*: the synthesis of the haptic device, virtual coupling, and virtual environment.

III. Terminology and Haptic Display Model

A. Terminology of Haptic Simulations

Figure 2 shows a diagram defining the various components of a *haptic simulation* [14]. In such a simulation, a *human operator* mechanically probes a *haptic device* to feel the dynamics of a *virtual environment*, a computer-generated model of a physically motivated scene. The haptic device and virtual environment are typically connected by an artificial *virtual coupling* to accommodate the complexity and non-linearity of exotic virtual environments while retaining control over the dynamics rendered to the operator.

The *haptic display* includes all components that determine the dynamics rendered to the human operator: the haptic device, virtual coupling, and virtual environment. Because we are primarily interested in describing the dynamics rendered to the human, we focus on the qualities of the haptic display.

B. Haptic Display Class

Haptic displays render force to the human from their control input in two fundamentally different ways. The nature of the map from a haptic display's *control input* to the rendered force determines the display's *class*. Figure 3 shows schematics of the different classes and causality structures.

1) Exogenous: Exogenous displays render force directly from the control input regardless of the motion of the haptic device. An example of an exogenous display is one actuated by an idealized motor modeled as a force source. Examples include the Touch™ and Phantom® Premium (3D Systems, Inc.) and omega.3 (Force Dimension) devices.

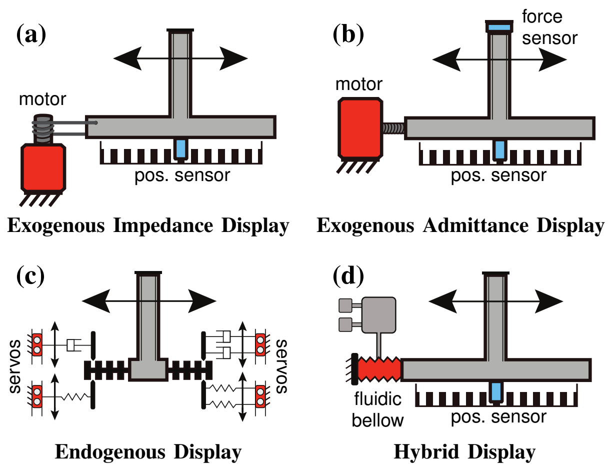


Fig. 3. Haptic displays of varying class and causality structure. Actuators are drawn in red and sensors in blue. (a) and (b) are both exogenous class displays: the control input renders force directly regardless of motion. (a) is impedance-type causality (motion determines force), and (b) is admittance-type causality (force determines motion). (c) is an endogenous class display: the control inputs modulate the relationship between motion and force. In this example, the springs and dampers can be connected/disconnected to user motion. (d) is a hybrid class display: the actuator is both exogenous and endogenous. In this example, a fluidic bellow controlled by pressure.

2) Endogenous: Endogenous displays render force from motion of the haptic device, and the relationship between motion and force is modulated by the control input. These displays cannot generate force without user-induced motion. An example of an endogenous display is one actuated by a spring that can be connected/disconnected to the device.

3) Hybrid: Hybrid displays render both force, and modulate the relationship between motion and force, from their control input. An example of a hybrid display is one actuated by a fluidic bellow where the control input is pressure.

C. Haptic Display Causality Structure

Haptic displays render dynamics to the human through a relationship between motion and force. A display's *causality structure* defines which is the input and output.

1) Impedance-Type: The display accepts velocity, (or more generally motion), and outputs force. Impedance-type displays are typically designed to be as mechanically transparent as possible, and render dynamics to the user through their haptic feedback loop. They can more easily render low stiffness and mass, but generally have less output force and dynamic range, than admittance-type displays.

2) Admittance-Type: The display accepts force and outputs velocity. Admittance-type displays are usually heavily geared and non-backdrivable by the human operator. The haptic feedback loop is used to dynamically compensate for the haptic device to allow motion. They can typically render high stiffness and mass, but struggle to render low impedances.

D. General Kinesthetic Haptic Display Model

Figure 4 presents a general haptic display model. The specifics of the class, causality structure, virtual coupling, virtual environment, and instrumentation error, are all abstracted into this representation. The haptic display

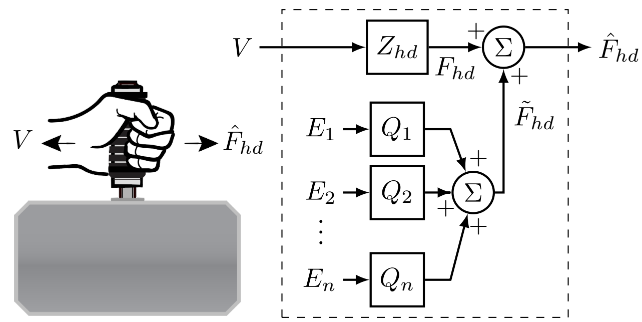


Fig. 4. General haptic display model: the display has impedance Z_{hd} and additive instrumentation errors $e_1, \ldots, e_n$, mapped to instrumentation force through $Q_1, \ldots, Q_n$. The arrows represent Laplace transforms of signals, for example, $\mathcal{L}\{\dot{v}(t)\} = \dot{V}(s)$, and boxes represent rational transfer functions, for example, $Z_{hd}(s)$.

impedance, $Z_{hd}(s)$, relates the input velocity, $v(t)$, to the display force, $f_{hd}(t)$. Additive instrumentation error signals, $e_1(t), e_2(t), \ldots, e_n(t)$ are mapped through transfer functions $Q_1(s), Q_2(s), \ldots, Q_n(s)$, to the total instrumentation error force, $\tilde{f}(t)$, where n represents the total number of error sources. The haptic display impedance is manipulated through a generalized control input u.

For simplicity, we present the haptic display dynamics as an impedance, $Z_{hd}(s)$. It is equally valid to present instead an admittance, $Y_{hd}(s)$; both representations can represent a haptic display of either causality structure.

IV. Z-Qualities

In this section we define the *Z-Qualities* (Figure 5) of a haptic display: characteristics that describe the display's stability, sensitivity to instrumentation error, speed of changing the rendered dynamics, and accuracy of the rendered dynamics to the desired dynamics. Given a specified set of Z-Qualities, it is possible to rigorously define if a haptic display can render particular dynamics with specified qualities.

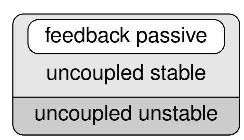
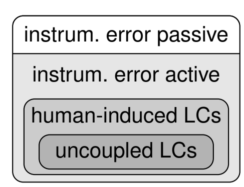
Z-Quality	Class Relationships
stability: $f(Z_{hd}) \mapsto S \in \mathcal{S}$ $\mathcal{S} = \{$ FBP = feedback passive, US = uncoupled stable $\}$	
instrumentation error: $f(Z_{hd}, e_1, ..., e_n) \mapsto E \in \mathcal{E}$ $\mathcal{E} = \{$ IEP = instrumentation error passive, HILC = human-induced limit cycles, ULC = uncoupled limit cycles $\}$	
speed: $f(Z_{hd}) \mapsto t^* \in \mathbb{R}+$	
accuracy: $f(Z_{hd}, Z_{des}) \mapsto \tilde{a}^* \in \mathbb{R}+$	

Fig. 5. The Z-Qualities describe a haptic display's stability, sensitivity to instrumentation error, speed of changing the rendered dynamics, and accuracy of the rendered dynamics to the desired dynamics. Given specified Z-Qualities, it is possible to rigorously define if a haptic display can render particular dynamics with specified characteristics.

A. Stability Qualities

The haptic display should be stable to prevent damage to the haptic device or injury to the human. Different stability qualities are required for different haptic rendering scenarios.

*1) **Feedback Passive, FBP**:* A haptic display is feedback passive if and only if the haptic display, with no instrumentation error, cannot generate energy at any time. Equivalently, when the haptic display is coupled to *any* passive human impedance, Z_h, the coupled haptic display and human operator system, Z_{sys}, is passive [15]. Feedback passivity implies bounded-input, bounded-output (BIBO) stability of Z_{sys}. Generally, feedback passivity can be conservative, or similar to, uncoupled stability depending on the haptic display control structure, virtual environment, and human coupling impedance. For exogenous, impedance-type, displays rendering stiffness and damping, feedback passivity's conservativeness is described by dimensionless inertia [15], and dimensionless damping [6]. For typical haptic displays and human impedances, feedback passivity is overly conservative as a practical coupled stability condition for exogenous impedance-type displays rendering stiffness and damping [6], or mass [16].

$$\text{feedback passive} \overset{\text{def}}{=} \int_0^t f(\tau)\, v(\tau)\, d\tau > 0 \quad \forall t,\ \text{admissible } f(t). \tag{1}$$

$$\begin{aligned} \text{feedback passive} \Leftrightarrow\ & Z_{sys} \text{ passive } \forall Z_h \,|\, Z_h \text{ passive}, \\ \Rightarrow\ & Z_{sys} \text{ stable } \forall Z_h \,|\, Z_h \text{ passive}. \end{aligned} \tag{2}$$

*2) **Uncoupled Stable, US**:* A haptic display is uncoupled stable if and only if the characteristic equation of $Z_{hd}(s)$ has no poles in the open right-half-plane, and only simple poles on the imaginary axis [14]. Uncoupled stability does not generally imply coupled stability with the range of dynamics of a human operator. Its use as a practical stability condition depends on the haptic display rendered dynamics and the coupling dynamics of the human operator. For exogenous impedance-type displays rendering stiffness and damping, uncoupled stability is a practical stability boundary for typical human interactions [3], [4], [5], [6]. However, in other haptic scenarios, such as exogenous admittance-type displays rendering free-space [7], or exogenous impedance-type displays rendering mass compensation [17], [16], uncoupled stability is not a practical stability boundary for common interactions.

$$\text{uncoupled stable} \overset{\text{def}}{=} Z_{hd} \text{ stable}. \tag{3}$$

B. Instrumentation Error Qualities

High-quality haptic rendering should be free of unnatural feeling impulses or vibrations that can be caused by instrumentation error. Additive instrumentation error cannot drive a system unstable in the BIBO sense. However, in some cases, the bounded impulses or oscillations are so large they limit the practical renderable dynamics of the haptic display.

The instrumentation error qualities of a haptic display are determined by the interplay of the energy dissipated by the haptic device, E_{diss}, and energy created from instrumentation energy, $\tilde{E}_{gen}$, over position trajectories and intervals of time. Of course, E_{diss} is specific to each haptic device. For example, for a haptic device modeled as a mass, viscous damper, and Coulomb friction,

$$E_{diss}(t_1 \to t_2) = \int_{t_1}^{t_2} \left(c\,\mathrm{sgn}(\dot{x}(\tau)) + b\dot{x}(\tau)\right)\, d\tau. \tag{4}$$

The instrumentation energy generated over an interval is

$$\tilde{E}_{gen}(t_1 \to t_2) = \int_{t_1}^{t_2} \tilde{f}(\tau)\, v(\tau)\, d\tau, \tag{5}$$

$$\text{where} \quad \tilde{f}(t) = \sum_{i=1}^{n} \left(\int q_i(\tau)\, e_i(t-\tau)\, d\tau \right), \tag{6}$$

and q_i is the impulse response of transfer function Q_i.

*1) **Instrumentation error passive, IEP**:* No net energy can be generated from instrumentation error over any time interval for arbitrary human interaction.

$$\text{IEP} \overset{\text{def}}{=} \nexists\, x(t), t_1, t_2 \,|\, \int_{t_1}^{t_2} \tilde{E}_{gen}(\tau) - E_{diss}(\tau)\, d\tau > 0. \tag{7}$$

A sufficient condition for instrumentation error passivity for impedance-type haptic displays rendering a general control law was generated by Colonnese and Okamura [6].

*2) **Human-induced limit cycles, HILC**:* There exist human induced trajectories of the haptic display for which it can have sustained bounded oscillations. In other words, it is possible to extract an unbounded amount of energy from the display due to instrumentation error.

$$\text{HILC} \overset{\text{def}}{=} \exists\, x(t) \,|\, \lim_{t\to\infty} \int_0^t \tilde{E}_{gen}(\tau) - E_{diss}(\tau)\, d\tau \quad \text{is unbounded}. \tag{8}$$

For exogenous impedance type displays rendering stiffness there exists a necessary and sufficient condition for HILC [8], [9], and a sufficient condition for HILC rendering stiffness and damping [6].

*3) **Uncoupled limit cycles, ULC**:* For certain initial conditions, the uncoupled (no human interaction) haptic display can have sustained oscillations, and thus, can persistently generate energy due to instrumentation error.

$$\text{ULC} \overset{\text{def}}{=} \exists\, x(t), t_0 \,|\, \lim_{t\to\infty} \int_{t_0}^t \tilde{E}_{gen}(\tau) - E_{diss}(\tau)\, d\tau \quad \text{is unbounded for } Z_h|_{t>t_0} = 0. \tag{9}$$

C. Z-speed

A high-quality haptic display should be able to change its rendered dynamics as quickly as the haptic simulation requires. The time it takes for the display to render a desired impedance, Z_{des}, within resolution Z_ϵ, is its Z-speed,

$$\text{Z-speed} \overset{\text{def}}{=} \arg\min_{t^*} \left(Z_{hd}(t)|_{t\geq t^*} = Z_{des} + Z_\epsilon \right). \tag{10}$$

Although not analyzed with the definition of Equation (10), researchers have explored various concepts of the speed of haptic displays rendering dynamics [10], [7].

D. Accuracy

In general, the rendered dynamics will be different than the desired dynamics. Describing and quantifying this difference is critical to describe the renderable range of a haptic display.

Griffiths et al. defined an accuracy concept called *distortion*, $\Theta(s)$, [11], which is the frequency dependent normalized difference between the rendered and desired dynamics, and showed an unavoidable tradeoff between accuracy and passivity. Colonnese et al. defined a decomposition of the rendered dynamics into *effective impedances*, mechanical primitives with physical analogs, and showed that exogenous impedance-type display rendering stiffness and damping only do so to finite bandwidth, and that rendered damping is extremely sensitive to time delay [12]. Also using effective impedances, Treadway and Gillespie showed excitation of high frequencies through contact transitions negatively impacts humans ability to distinguish between stiffnesses [13]. Accuracy concepts such as distortion and effective impedances are useful to comprehensively describe accuracy, however, because of their dimensionality, are not suitable as a Z-Quality. A straightforward scalar concept of accuracy is the Average Distortion Error, [12],

$$\text{ADE} \stackrel{\text{def}}{=} \int_0^\infty W(\omega)\, \frac{|Z_{hd}(j\omega) - Z_{des}(j\omega)|}{|Z_{des}(j\omega)|}\, d\omega, \quad (11)$$

where $\int_0^\infty W(\omega) = 1$, and $W(\omega) \geq 0 \,\forall\, \omega$. The weighting function, $W(\omega)$, is a parameter that represents the relative importance of the frequencies of the haptic display rendering.

V. Renderable Mass-Damping-Stiffness Spaces

The Z-qualities provide a framework to answer if a given haptic display can render *specific* dynamics with certain qualities. It would be useful to define a *set* of standard dynamics to easily communicate what a haptic display can render. To address this, we propose the *Renderable Mass-Damping-Stiffness, MBK, Spaces* of a haptic display: the set of all mass-damper-spring impedances that the display can render that satisfy specified Z-Quality constraints. This three dimensional representation, where each coordinate represents dynamics with certain mass, damping, and stiffness, is chosen because it can describe many of the desired dynamics for typical haptic scenarios, and because it can be shown as a volume. Of course, because there are infinite possible Z-Qualities, there are also infinite renderable MBK spaces.

Formally, the MBK space of a haptic display, MBK(Z_{hd}), for specified Z-qualities (Figure 5) of stability, $S \in \mathcal{S}$, instrumentation error, $E \in \mathcal{E}$, speed, t^*, and accuracy error, $\tilde{a}^*$, is

$$\begin{aligned} \text{MBK}\,(Z_{hd}) &\stackrel{\text{def}}{=} \{Z_{mbk,i}\} \text{ for } i = 1, 2, \ldots, p \\ &\text{where } \exists\, u_i, m_i, b_i, k_i \\ &\text{such that } Z_{hd}(u_i) \in S \\ &\qquad Z_{hd}(u_i, e_1, e_2, \ldots, e_n) \in E \\ &\qquad \text{Z-speed}\,(Z_{hd}(u_i)) \leq t^* \\ &\qquad \text{ADE}\,(Z_{hd}(u_i), Z_{mbk,i}) \leq \tilde{a}^*, \end{aligned} \quad (12)$$

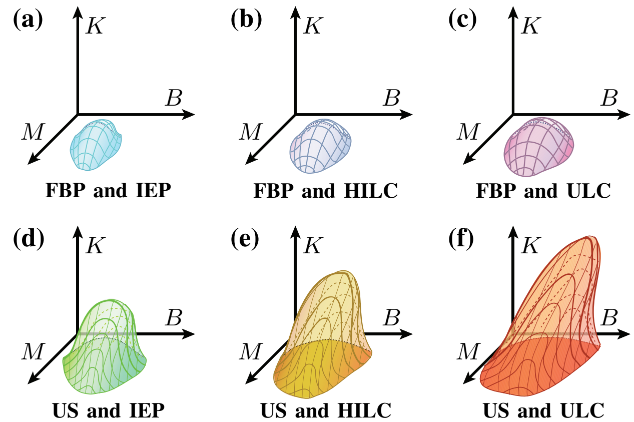


Fig. 6. Illustrative MBK spaces of a haptic display for unchanging speed and accuracy Z-Qualities, but different stability and instrumentation error Z-qualities (see Figure 5 for definitions). As the conditions for stability and instrumentation error relax, the corresponding MBK spaces expand. Note that because of the set relationships for stability and instrumentation error: a ⊆ b, b ⊆ c, d ⊆ e, e ⊆ f, a ⊆ d, b ⊆ d, and c ⊆ f.

where u is the generalized control input, p is the (possibly infinite) mass, m_i, damping, b_i, and stiffness, k_i, mass-damper-spring impedance parameters, and

$$Z_{mbk,i} = m_i s + b_i + k_i/s. \quad (13)$$

A given haptic display has many MBK spaces. Figure 6 shows illustrative examples of various MBK spaces for a single haptic display. Each of these MBK spaces have identical speed and accuracy, but different stability and instrumentation error, Z-Qualities conditions.

Of all the possible Z-Qualities specifications, two are particularly popular for common haptic display applications:

$$\begin{aligned} \text{Passiva} &= \{S = \text{FBP},\ E = \text{IEP},\ t^* \leq 1\,\text{ms},\ \tilde{a}^* \leq 1\}, \\ \text{Stabila} &= \{S = \text{US},\ \ E = \text{IEP},\ t^* \leq 1\,\text{ms},\ \tilde{a}^* \leq 1\}. \end{aligned}$$

They are commonly desired Z-Qualities because a) feedback passivity or uncoupled stability is the practical robust stability condition for the haptic scenario b) instrumentation error passivity is required because humans are incredibly sensitive to haptic impulses/vibrations, c) fast switching of the rendered dynamics in needed for high quality, and d) accurate rendering of desired dynamics is essential. Of course, suitable Z-Qualities specifications will vary for each haptic scenario. Figure 7 shows illustrative examples of MBK spaces corresponding to the Passiva and Stabila Z-Qualities for three different types of haptic displays.

VI. Conclusion and Future Work

In this paper we a presented a framework for defining and communicating the renderable dynamics of a general kinesthetic haptic display. We hope it is a useful benchmarking and communication tool, especially for haptic scenarios with stringent conditions for the dynamics rendered to the human.

In the future we plan to generate experimental MBK spaces for popular Z-Qualities, haptic devices, control approaches, and virtual environment rendering schemes. We imagine an online database of the MBK spaces, created in the aim of stimulating and aiding haptic display research.

TABLE I
CURRENT STATE OF RESEARCH IN Z-QUALITIES

	Stability		Instrum. Error			Speed	Accur.	Class	Caus.	Control Law			
	FBP	US	IEP	HILC	ULC					G	K	B	M
Colgate 1994, 1997 [2], [15]	✓							Exo	Imp	✓	✓	✓	
Adams 1999 [14]	✓							Exo	Both	✓	✓	✓	
Abbott 2005 [8], Diolaiti 2006 [9]	✓		✓	✓				Exo	Imp		✓		
Griffiths 2008, 2011 [18], [11]	✓						✓	Exo	Both	✓	✓	✓	✓
Gil 2009 [17]		✓					✓	Exo	Imp				✓
Gil, Díaz, Ciáurriz 2009-2014 [3], [4], [19]		✓						Exo	Imp		✓	✓	
Hulin 2013 [10]	✓	✓				✓		Exo	Imp		✓	✓	
Hulin 2014 [5]	✓	✓						Exo	Imp		✓	✓	
Colonnese 2015 [16]	✓	✓	✓	✓	✓			Exo	Imp	✓	✓	✓	
Colonnese 2015 [12]	✓	✓			✓		✓	Exo	Imp				✓
Colonnese 2016 [6]							✓	Exo	Imp	✓	✓	✓	
Treadway 2017, 2018 [20], [13]							✓	Exo	Both		✓	✓	✓
Parthiban 2018 [7]		✓				✓	✓	Exo	Adm		✓	✓	✓

FBP = feedback passive, US = uncoupled stable, IEP = instrumentation error passive, HILC = human-induced limit cycles, ULC = uncoupled limit cycles, Exo = exogenous, Imp = impedance, Adm = admittance, G = general, K = stiffness, B = damping, and M = mass.

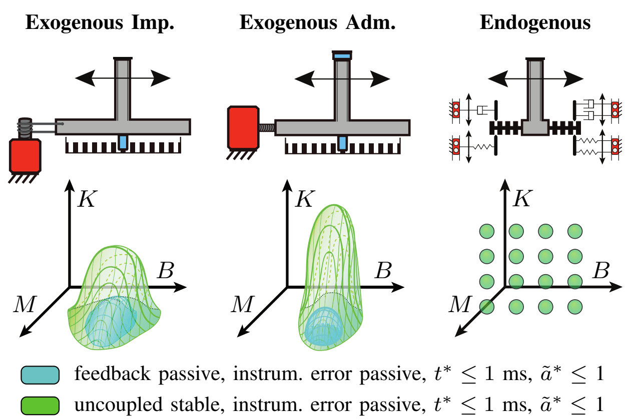


Fig. 7. Illustrative MBK spaces corresponding to the Passiva and Stabila Z-Qualities for different types of haptic displays. These illustrative examples show the general benefits and drawbacks of each type of display. Exogenous impedance-type displays excel at rendering low impedances, and exogenous admittance-type displays, at rendering high impedances. Endogenous displays do not have problems with stability or instrumentation error, but can render only select dynamics specific to the design of the display.

REFERENCES

[1] R. Hinchet, V. Vechev, H. Shea, O. Hilliges, and E. Zurich, "DextrES: Wearable haptic feedback for grasping in VR via a thin form-factor electrostatic brake," in *Proc. ACM UIST*, 2018, pp. 901–912.

[2] J. Colgate and J. Brown, "Factors affecting the z-width of a haptic display," in *Proc. IEEE Int. Conf. Robot. Automat.*, 1994, pp. 3205–3210.

[3] J. J. Gil, E. Sánchez, T. Hulin, C. Preusche, and G. Hirzinger, "Stability boundary for haptic rendering: Influence of damping and delay," *J. Computing and Information Science in Engineering*, vol. 9, no. 1, pp. 011 005–1–011 005–8, 2009.

[4] I. Díaz and J. J. Gil, "Influence of vibration modes and human operator on the stability of haptic rendering," *IEEE Trans. Robotics*, vol. 26, no. 1, pp. 160–165, 2010.

[5] T. Hulin, A. Albu-Schäffer, and G. Hirzinger, "Passivity and stability boundaries for haptic systems with time delay," *IEEE Trans. Contr. Syst. Technol.*, vol. 22, no. 4, pp. 1297–1309, 2014.

[6] N. Colonnese and A. Okamura, "Stability and quantization-error analysis of haptic rendering of virtual stiffness and damping," *Int. J. Robotics Research*, vol. 35, no. 9, pp. 1103–1120, 2016.

[7] C. Parthiban and M. Zinn, "Performance and stability limitations of admittance-based haptic interfaces," in *Proc. IEEE Haptics Symp.*, 2018, pp. 58–65.

[8] J. J. Abbott and A. M. Okamura, "Effects of position quantization and sampling rate on virtual-wall passivity," *IEEE Trans. Robotics*, vol. 21, no. 5, pp. 952–964, 2005.

[9] N. Diolaiti, G. Niemeyer, F. Barbagli, and J. K. Salisbury, "Stability of haptic rendering: Discretization, quantization, time delay, and coulomb effects," *IEEE Trans. Robotics*, vol. 22, no. 2, pp. 256–268, 2006.

[10] T. Hulin, R. G. Camarero, and Albu-Schäffer, "Optimal control for haptic rendering: Fast energy dissipation and minimum overshoot," in *Proc. IEEE/RSJ Int. Conf. Intelligent Robots and Systems*, 2013, pp. 4505–4511.

[11] P. G. Griffiths, R. B. Gillespie, and J. S. Freudenberg, "A fundamental linear systems conflict between performance and passivity in haptic rendering," *IEEE Trans. Robotics*, vol. 27, no. 1, pp. 75–88, 2011.

[12] N. Colonnese, A. F. Siu, C. M. Abbott, and A. M. Okamura, "Rendered and characterized closed-loop accuracy of impedance-type haptic displays," *IEEE Trans. Haptics*, vol. 8, no. 4, pp. 434–446, 2015.

[13] E. Treadway and R. B. Gillespie, "Unilateral and bilateral virtual springs: Contact transitions unmask device dynamics," *IEEE Trans. Haptics*, 2018.

[14] R. J. Adams and B. Hannaford, "Stable haptic interaction with virtual environments," *IEEE J. Robot. Automat.*, vol. 15, no. 3, pp. 465–474, 1999.

[15] J. E. Colgate and G. G. Schenkel, "Passivity of a class of sampled-data systems: Application to haptic interfaces," *J. of Robotic Systems*, vol. 14, no. 1, pp. 37–47, 1997.

[16] N. Colonnese and A. M. Okamura, "M-Width: Stability, noise characterization, and accuracy of rendering virtual mass," *Int. J. Robotics Research*, vol. 34, no. 6, pp. 781–798, 2015.

[17] J. J. Gil, Á. Rubio, and J. Savall, "Decreasing the apparent inertia of a haptic device by using force feedforward," *IEEE Trans. Contr. Syst. Technol.*, vol. 17, no. 4, pp. 833–838, 2009.

[18] P. G. Griffiths, R. B. Gillespie, and J. S. Freudenberg, "A fundamental tradeoff between performance and sensitivity within haptic rendering," *IEEE Trans. Robotics*, vol. 24, no. 3, pp. 537–548, 2008.

[19] P. Ciaurriz, I. Diaz, and J. J. Gil, "Stable discrete-time impedances for haptic systems with vibration modes and delay," *IEEE Trans. Contr. Syst. Technol.*, vol. 22, no. 3, pp. 884–895, 2014.

[20] E. Treadway, Y. Yang, and R. B. Gillespie, "Decomposing the performance of admittance and series elastic haptic rendering architectures," in *Proc. IEEE World Haptics Conf.*, no. June, 2017, pp. 346–351.